

University of California, Riverside
Department of Mathematics

Final Exam
Mathematics 9B - First Year of Calculus
Sample 5

Instructions: This exam has a total of 140 points. You have 3 hours. You must show all your work to receive full credit. You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed to use books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

1. Evaluate the following integrals.

(a) (6 points)

$$\int_1^8 \sqrt[3]{x} \, dx$$

(b) (7 points)

$$\int \frac{x^2}{(1+x^3)^2} \, dx$$

(c) (7 points)

$$\int_1^e \frac{\cos(\ln(x))}{x} \, dx$$

2. (20 points) Find the area of the region enclosed by the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

3. (20 points) Consider the region enclosed by the curve $y = \sqrt{1 + e^x}$, and the lines $x = 0$, $x = 1$ and $y = 0$. Find the volume of the solid obtained by rotating the region about the x -axis.

4. Use the L'Hospital's Rule to evaluate the following limits.

(a) (10 points)

$$\lim_{x \rightarrow 0} \frac{\sin^{-1}(x)}{x}$$

(b) (10 points)

$$\lim_{x \rightarrow 0} (1 + 2x)^{\frac{1}{x}}$$

5. Find the following integrals.

(a) (6 points)

$$\int x \cos(x) dx$$

(b) (7 points)

$$\int \tan^{-1}(x) \cdot (3x^2 + 1) dx$$

(c) (7 points)

$$\int \sin^3(x) \cos^2(x) dx$$

6. Find the following integrals.

(a) (10 points)

$$\int \frac{3x - 1}{2x^2 - x} dx$$

(b) (10 points)

$$\int \frac{\sqrt{x+1}}{x} dx$$

7. Answer the following questions.

(a) (10 points) Evaluate the integral.

$$\int_1^{\infty} \frac{1}{x^3} dx$$

(b) (10 points) Does the following integral converge or diverge? Prove your answer!

$$\int_1^{\infty} \frac{\sin^2(x)}{x^3} dx$$