

University of California, Riverside
Department of Mathematics

Final Exam
Mathematics 9B - First Year of Calculus
Sample 4

Instructions: This exam has a total of 140 points. You have 3 hours. You must show all your work to receive full credit. You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed to use books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

1. (a) (6 points) State the Fundamental Theorem of Calculus.
(b) (7 points) Evaluate the integral

$$\int_0^1 \frac{d}{dx} \left(e^{\tan^{-1}(x)} \right) dx$$

- (c) (7 points) Compute

$$\frac{d}{dx} \int_1^{\frac{1}{x}} \sin t dt$$

2. (20 points) Find the volume of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the line $y = 2$.
3. (20 points) An aquarium 3m long, 6m wide and 1m deep is full of water. Find the work (in Joule) needed to pump half of the water out of the aquarium. (Use the facts that the density of the water is 1000kg/m^3 and $g = 9.8\text{m/sec}^2$.)
4. (a) (10 points) Find a formula for the inverse of the function

$$f(x) = \frac{4x - 1}{2x + 3}$$

- (b) (10 points) Find $(f^{-1})'(4)$ for the function

$$f(x) = 4 + 3x^2 + \tan\left(\frac{\pi x}{2}\right), \quad -1 < x < 1$$

5. Find the following limits:

(a) (10 points)

$$\lim_{x \rightarrow \infty} \frac{\ln(2x)}{\sqrt{2x}}$$

(b) (10 points) Use the definition of the function $y = \tanh x$ to compute

$$\lim_{x \rightarrow \infty} \tanh(x)$$

6. Evaluate the following integrals:

(a) (6 points)

$$\int \frac{dx}{x^2 \sqrt{x^2 - 16}}$$

(b) (7 points)

$$\int_{-\pi}^{\pi} \sin^3 x \cos^3 x dx$$

(c) (7 points)

$$\int_0^1 \frac{x - 3}{x^2 + 6x + 5} dx$$

7. Evaluate the following integrals or show that they are divergent:

(a) (10 points)

$$\int_1^{\infty} \frac{\ln x}{x^4} dx$$

(b) (10 points)

$$\int_0^1 \frac{3 \ln x}{\sqrt{x}} dx$$