University of California, Riverside
Department of Mathematics

Final Exam
Mathematics 9B - First Year of Calculus
Sample 2

Instructions: This exam has a total of 140 points. You have 3 hours. You must show all your work to receive full credit. You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed to use books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

1. Consider the region bounded by the following two functions:

$$y = 2(-x^2 + 9) \ \text{ and } \ y = 0.$$

   (a) (6 points) Using the lower sum with three rectangles having equal width, approximate the area.

   (b) (7 points) Using the upper sum with three rectangles having equal width, approximate the area.

   (c) (7 points) Find the actual area of the region.

2. Consider the area bounded by the following two functions:

$$y = \sin x \ \text{ and } \ y = \frac{2}{\pi}x.$$

   (a) (10 points) Find the three intersection points of the two given functions. (Drawing may be helpful.)

   (b) (10 points) Find the area bounded by the two functions.

3. Compute the following integrals.

   (a) (6 points) $\int e^x (x + \sin(e^x)) dx$

   (b) (7 points) $\int \frac{2x^2 + 1}{2x^2 + x} dx$

(c) (7 points) $\int \sin^3 x dx$

4. Consider the solid obtained by rotating the area bounded by the following three functions about y-axis:

$$x = 0, \quad y = e^x, \quad \text{and} \quad y = ex.$$

   (a) (6 points) Sketch the region bounded by the given three functions. Find the intersection point of the two functions: $y = e^x$ and $y = ex$. (There is only one.)

   (b) (7 points) Set up the integral for the **volume** of the solid.

   (c) (7 points) Find the volume of the solid by computing the integral.

5. Evaluate the following limits:

   (a) (7 points) $\lim_{x \to \infty} \frac{x^3}{e^x}$

   (b) (6 points) $\lim_{x \to 1} \frac{x^3 - 1}{x^{10} - 1}$

   (c) (7 points) $\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{5x}$

6. A rock climber starts to climb a wall of height 50 ft and rope attached to her. The rope weighs t^2 lb/ft at the point t ft away from the end attached to her. (The rope is longer than 50 ft.)

   (a) (10 points) When the climber is x ft above the ground, how much does the rope weigh?

   (b) (10 points) While climbing to the top, how much work does she do?

7. Consider the function $f(x) = y = \tan^3 x + 3$.

   (a) (6 points) Find the maximal domain containing $x = 0$ where the function $f(x)$ is one-to-one. Explain why $f(x)$ is one-to-one on that domain.

   (b) (7 points) If $g(x)$ in the inverse of $f(x)$, find the domain and the range of $g(x)$.

   (c) (7 points) Find $g'(4)$.