University of California, Riverside
Department of Mathematics

Midterm
Mathematics 9B - First Year of Calculus
Sample 1

Instructions: This exam has a total of 100 points. You have 50 minutes. You must show all your work to receive full credit You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

1. Evaluate the indefinite and definite integrals.

   (a) (10 points)
   $$\int x^2 \sqrt{1 + x^3} \, dx$$

   (b) (10 points)
   $$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos(x)}{\sin^2(x)} \, dx$$

2. (20 points) Find the average value of the function on the given interval.
   $$f(x) = 2x^3(1 + x^2)^4, \quad [0, 2]$$

3. (20 points) Compute the area of the region in the first quadrant bounded on the left by the curve $y = \sqrt{x}$, on the right by the curve $y = 6 - x$, and below by the curve $y = 1$.

4. (20 points) A region in the first quadrant is bounded by the curves $y = x^2$, $y = 2 - x^2$ and $x = 0$. Compute the volume of the solid obtained by rotating this region about the line $x = 1$. You may use any of the three methods (disk, washer or shell) which you deem appropriate.

5. Let $f(x) = 1 - x^2$.

   (a) (7 points) Compute the left-hand Riemann sum approximation of $\int_0^3 f(x)\,dx$ with $n = 3$ boxes.

   (b) (7 points) Compute the right-hand Riemann sum approximation of $\int_0^3 f(x)\,dx$ with $n = 3$ boxes.

   (c) (6 points) Express $\int_0^3 f(x)\,dx$ as a limit of right-hand Riemann sums (as in the definition of the definite integral). Do not evaluate the limit.