University of California, Riverside
Department of Mathematics

Final Exam

Mathematics 9B - First Year of Calculus
Sample 1

Instructions: This exam has a total of 140 points. You have 3 hours. You must show all your work to receive full credit. You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed to use books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

1. We would like to evaluate
$$\frac{d}{dx}\left(\int_{-1}^{x}\sin(t^2)2tdt\right).$$
   (a) (5 points) Compute $f(x)=\int_{-1}^{x}\sin(t^2)2tdt$
   (b) (5 points) Find $f'(x)$.
   (c) (5 points) State the fundamental theorem of calculus.
   (d) (5 points) Use the fundamental theorem of calculus to compute $\frac{d}{dx}\left(\int_{-1}^{x}\sin(t^2)2tdt\right)$ without first computing the integral.

2. (20 points) Find the area of the region between the two curves $y=3x-x^2$ and $y=2x^3-x^2-5x$.

3. (a) (10 points) Set up an integral to calculate the volume of the solid obtained by rotating the region between $y=5-x$ and $y=25-x^2$ around the x-axis.
   (b) (10 points) Evaluate the integral.

4. Evaluate the following definite integrals.

(a) (4 points)

$$\int_0^2 \left(2x^2 - 4x + 1\right)^4 (x - 1)\, dx$$

(b) (4 points)

$$\int_0^{\frac{\pi}{4}} \cos^3 x \sin^4 x dx$$

(c) (4 points)

$$\int_{\sqrt{3}}^{\sqrt{11}} \frac{x}{\sqrt{x^2 - 2}} dx$$

(d) (4 points)

$$\int_1^2 e^{x^4 - 3x + 1}(4x^3 - 3) dx$$

(e) (4 points)

$$\int_{-4}^0 \frac{x}{x + 5} dx$$

5. Evaluate the following indefinite integrals:

(a) (4 points)

$$\int \left(9x^4 - 3x^3 + 7\right)^{50} \left(4x^3 - x^2\right) dx$$

(b) (4 points)

$$\int e^{x^4 + 2x^2}(x^3 + x) dx$$

(c) (4 points)

$$\int x^5 \ln x dx$$

(d) (4 points)

$$\int \frac{3}{\sqrt{16 - x^2}} dx$$

(e) (4 points)

$$\int \frac{8x + 6}{9 + x^2} dx$$

6. Find the derivative of each of the following functions:

   (a) (10 points) $y = 8^{3x-1}$

   (b) (10 points) $y = \ln(4x^5 - \tan x)$.

7. The function $f(x) = x^3 + 6x^2 + 14x - 6$ has an inverse function $g(x) = f^{-1}(x)$. (We know this because $f(x)$ is one-to-one; you don't need to solve for $g(x)$.)

   (a) (6 points) Find a formula for $g'(x)$.

   (b) (7 points) Since $f(1) = 15$, what is $g(15)$?

   (c) (7 points) Find $g'(15)$ from the formula in (a).