University of California, Riverside
Department of Mathematics

Midterm 2
Mathematics 8B - First Year of Calculus
Sample 5

Instructions: This exam has a total of 100 points. You have 50 minutes. You must show all your work to receive full credit You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

Show all your calculations in detail. Explain and justify every step. .

1. Define the function

$$f(x) = \begin{cases} -x + 1, & x < 2, \\ x^2 - 6x + 5, & x \geq 2. \end{cases}$$

   (a) (7 points) Find the right hand sided limit $\lim_{x \to 2^+} f(x)$.

   (b) (7 points) Find the left hand sided limit $\lim_{x \to 2^-} f(x)$.

   (c) (6 points) Is $f(x)$ continuous at $x = 2$?

2. Evaluate the following limits:

   (a) (6 points)
$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$$

   (b) (7 points)
$$\lim_{x \to 0} \frac{\sin(5x)}{3x}.$$

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(c) (7 points) Use the Sandwich Theorem to find $\lim\limits_{x \to 0} x^2 \cos \frac{1}{x}$.

3. (20 points) Use the Intermediate Value Theorem to show that $f(x) = x^3 + x - 5$ has at least a root on the interval $[0, 2]$.

4. (a) (5 points) Use the definition of derivative to find $f'(x)$ for $f(x) = \sqrt{x}$.

   (b) (5 points) Find the tangent line of the graph of $f(x)$ at $(4, 2)$.

   (c) (10 points) Find the derivative of $f(x) = (x^2 + x)\sin(x) + \frac{\cos(x)}{x+1}$.

5. Let $f(x) = 2x + 1$.

   (a) (5 points) Find $L = \lim\limits_{x \to 1} f(x)$.

   (b) (15 points) For a given $\varepsilon > 0$ find a $\delta > 0$, such that $0 < |x-1| < \delta$ implies $|f(x) - L| < \varepsilon$.