University of California, Riverside
Department of Mathematics

Final Exam

Mathematics 8B - First Year of Calculus
Sample 5

Instructions: This exam has a total of 200 points. You have 3 hours. You must show all your work to receive full credit You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

The following formulae may be useful: for any A, B, we have

$$\begin{aligned}
\sin(A+B) &= \sin(A)\cos(B)+\cos(A)\sin(B),\\
\cos(A+B) &= \cos(A)\cos(B)-\sin(A)\sin(B),\\
\tan(A+B) &= \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}, \qquad \tan\left(\frac{A}{2}\right)=\frac{\sin(A)}{1+\cos(A)}=\frac{1-\cos(A)}{\sin(A)},\\
\sin\left(\frac{A}{2}\right) &= \pm\sqrt{\frac{1-\cos(A)}{2}}, \qquad \cos\left(\frac{A}{2}\right)=\pm\sqrt{\frac{1+\cos(A)}{2}},\\
\sin(2A) &= 2\sin(A)\cos(A), \qquad \cos(2A)=\cos^2(A)-\sin^2(A),\\
\cos(2A) &= 2\cos^2(A)-1, \qquad \cos(2A)=1-2\sin^2(A),\\
\tan(2A) &= \frac{2A}{1-\tan^2(A)}
\end{aligned}$$

1. This question has three parts

   (a) (5 points) Find $\arcsin\left(\sqrt{3}/2\right)$,

   (b) (5 points) Find $\tan\frac{7\pi}{12}$,

   (c) (10 points) Suppose that $\sin(\theta)=3/4$ and that θ is in Quadrant II. Find $\cos(\theta)$ and $\cot(\theta)$

2. This question concerns limits.

   (a) (10 points) Compute $\lim_{x\to 0} \frac{\sin(2x)}{3x}$

   (b) (10 points) Compute $\lim_{x\to\infty} \frac{x^2+1}{3x^3-4x+1}$

3. This question concerns derivatives

   (a) (10 points) Using only the definition (i.e. using limits) of a derivative, compute the derivative of $\sqrt{x}$.

   (b) (10 points) Using the rules for taking derivatives, find the derivative of $f(x) = \sin(\cos(2x+1))$

4. Suppose that $y^6 + x^2y + x = 3$

   (a) (10 points) Implicitly differentiate to find y' in terms of x and y.

   (b) (10 points) Find the tangent line to the curve given by $y^6 + x^2y + x = 3$ at the point $P(1,1)$.

5. Suppose a spherical ballon expands in radius at 1 foot/sec, and suppose we consider the instant of time t_0 when the radius is 2 feet.

   (a) (8 points) What is the volume of the sphere at this time?

   (b) (12 points) By what amount is the volume expanding at this instant of time?

6. (20 points) Find two positive real numbers with smallest possible sum, whose product is 25.

7. Suppose $p(x) = x^3 + 3x + 3$

   (a) (10 points) Using the Intermediate Value Theorem, show that p has a root in $[-1, 0]$.

   (b) (10 points) Now use the Rolle's Theorem to show that p has only one root in $\Re$.

8. Let $f(x) = x^3 + x$. Compute its

   (a) (3 points) domain

(b) (3 points) range

(c) (3 points) symmetries (odd, even, both, neither)

(d) (3 points) horizontal asymptotes

(e) (3 points) vertical asymptotes

(f) (3 points) first two derivatives

(g) (3 points) critical points and points of inflection

(h) (9 points) Now draw the graph of $f(x)$, including intercepts, local maxima and local minima, points of inflections, and areas where the graph is concave up or concave down.

9. (20 points) Suppose a box with a square base and open top has a volume of 32 cubic feet. Find the dimensions of the box that minimize the amount of the material used.

10. (10 points) Find the most general antiderivative of $f(x) = \frac{10x^2+2x}{x^5}$