University of California, Riverside
Department of Mathematics

# Midterm 2
## Mathematics 8B - First Year of Calculus
### Sample 4

Instructions: This exam has a total of 100 points. You have 50 minutes. You must show all your work to receive full credit You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

**Show all your calculations in detail. Explain and justify every step.**

1. Suppose that
$$f(x) = \begin{cases} x^2 - 4 & \text{if } x < 2 \\ x^2 + 4 & \text{if } x \geq 2 \end{cases}$$

   (a) (7 points) Find
   $$\lim_{x \to 2^-} f(x)$$

   (b) (7 points) Find
   $$\lim_{x \to 2^+} f(x)$$

   (c) (6 points) Is $f(x)$ continuous at $x = 2$?

2. Compute the following limits, and state any theorems that you use:

   (a) (10 points)
   $$\lim_{x \to 2} \frac{(x-2)^2}{x^2 - 4}$$

(b) (10 points)
$$\lim_{x \to 0} \frac{\sin 4x}{3x}$$

3. This question concerns derivatives.

   (a) (10 points) Find the derivative of $y = \sqrt{x} + x \cos x$.

   (b) (10 points) Find the equation of the tangent line to $y = x^3$ at $x = 1$.

4. This question concerns to Intermediate Value Theorem.

   (a) (10 points) State the Intermediate Value Theorem.

   (b) (10 points) Now use this theorem to prove that $x^3 - x + 1$ has a root between $-2$ and $0$.

5. Suppose a man is driving away from Riverside, with his position at $t$ is given by the function
$$d(t) = \frac{t^2}{t^4 + 1}$$

   (a) (6 points) Compute his velocity at time $t$. (You may use differentiation formulae here; you do NOT need to compute the limit of the difference quotient.)

   (b) (7 points) At what times is he at rest?

   (c) (7 points) When is he moving forwards? When is he moving backwards?