University of California, Riverside
Department of Mathematics

Midterm 2
Mathematics 8B - First Year of Calculus
Sample 2

Instructions: This exam has a total of 100 points. You have 50 minutes. You must show all your work to receive full credit You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

Show all your calculations in detail. Explain and justify every step.

1. (20 points) Prove that the polynomial $p(x) = x^3 - 5x - 8$ has a root in the interval $[2, 3]$.

2. (20 points) Define the function f by

$$f(x) = \begin{cases} x \cot (3x) & x \neq 0 \\ \alpha & x = 0 \end{cases}$$

   Find the value of α such that $f(x)$ is continuous at $x = 0$.

3. Evaluate the following limits:

   (a) (10 points)
   $$\lim_{x \to 2} \frac{3(x^2 - 4)}{x^2 + x - 6}$$

   (b) (10 points)
   $$\lim_{x \to 0} x^2 \sin \frac{1}{x}$$

4. (20 points) Calculate the derivative $f'(x)$ of

$$f(x) = \frac{x(2-x)}{x^2 + 3}$$

5. (20 points) Use the definition of the derivative to calculate $f'(x)$ of the function $f(x) = \sqrt{x-3}$. Find the slope of the tangent line to the graph of $f(x)$ at the point $x = 4$.