University of California, Riverside
Department of Mathematics

Final Exam

Mathematics 8B - First Year of Calculus
Sample 2

Instructions: This exam has a total of 200 points. You have 3 hours. You must show all your work to receive full credit You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

The following formulae may be useful: for any A, B, we have

$$\begin{aligned}
\sin(A+B) &= \sin(A)\cos(B)+\cos(A)\sin(B), \\
\cos(A+B) &= \cos(A)\cos(B)-\sin(A)\sin(B), \\
\tan(A+B) &= \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}, \qquad \tan\left(\frac{A}{2}\right)=\frac{\sin(A)}{1+\cos(A)}=\frac{1-\cos(A)}{\sin(A)}, \\
\sin\left(\frac{A}{2}\right) &= \pm\sqrt{\frac{1-\cos(A)}{2}}, \qquad \cos\left(\frac{A}{2}\right)=\pm\sqrt{\frac{1+\cos(A)}{2}}, \\
\sin(2A) &= 2\sin(A)\cos(A), \qquad \cos(2A)=\cos^2(A)-\sin^2(A), \\
\cos(2A) &= 2\cos^2(A)-1, \qquad \cos(2A)=1-2\sin^2(A), \\
\tan(2A) &= \frac{2A}{1-\tan^2(A)}
\end{aligned}$$

1. Evaluate the following values:

   (a) (6 points) Find $\arcsin(-\frac{1}{2})$.

   (b) (6 points) Find $\tan(\frac{5\pi}{4})$

   (c) (8 points) If α is in the second quadrant with $\sin\alpha = \frac{1}{2}$, what is $\cos\alpha$?

2. Find the following limits

(a) (6 points)

$$\lim_{x \to 3} \frac{x^2 - 5x + 6}{x - 3}$$

(b) (6 points)

$$\lim_{x \to 0} \frac{\sin(x^2)}{3x^2}.$$

(c) (8 points) $\lim_{x \to +\infty} (\sqrt{x^2 + 1} - x)$.

3. This problem is asking you to compute derivatives:

   (a) (10 points) Use the definition of derivative to find $f'(x)$ for $f(x) = \sqrt{x + 2}$.

   (b) (10 points) Use any method to find the derivative of $f(x) = (x + \sin x)^2$.

4. Let $f(x) = x^3 + 2x + 1$.

   (a) (10 points) Use the Intermediate Value Theorem to show $f(x)$ has at least one root on the interval $[-2, 1]$.

   (b) (10 points) Use the Mean Value Theorem to show that $f(x)$ has exactly one root on the interval $[-2, 1]$.

5. (10 points) Given $x^2 + y^3 = 5$, find $\frac{dy}{dx}$.

6. Given $f(x) = x^{\frac{1}{3}}(x + 4)$.

   (a) (10 points) Find the critical points of $f(x)$ on $[-8, 1]$.

   (b) (10 points) Find the absolute maximum and minimum value of $f(x)$ on the above interval.

7. (20 points) A 25−ft ladder is leaning against a house when its base starts of slide away. By the time the base is 20−ft from the house, the base is moving at the rate of 6−ft/sec. How fast is the top of the ladder sliding down the wall then

8. (20 points) A farmer has 24 ft of fencing and wants to fence o ff a rectangular field that borders a river. What are the dimensions of the field that has largest area?

9. $f(x) = x^4(x-5)$.

   (a) (10 points) Find the critical points of the function and determine the intervals of increase and decrease.

   (b) (10 points) Determine the intervals on which the function concave up and down and find the inflection points.

   (c) (10 points) Find the local maxima and minima if there is any.

10. Find the most general antiderivative of the following functions:

   (a) (10 points) $f(x) = \sin 3x + x^2$

   (b) (10 points) $g(x) = 3\sqrt{x} + \frac{4}{x^{\frac{1}{3}}}$