

University of California, Riverside  
Department of Mathematics

**Midterm 2**  
Mathematics 8B - First Year of Calculus  
Sample 1

Instructions: This exam has a total of 100 points. You have 50 minutes. You must show all your work to receive full credit. You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed books, notes, or calculators. Answers should be written as  $\sqrt{2}$  as opposed to 1.4142135....

**Show all your calculations in detail. Explain and justify every step.**

1. (a) (10 points) Determine  $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$ .  
(b) (10 points) Determine  $\lim_{x \rightarrow 2^+} f(x)$ , if  $f(x) = \begin{cases} x + 2 & x \leq 2 \\ x^2 & 2 < x \leq 3 \end{cases}$
2. (a) (10 points) Carefully state the “ $\varepsilon/\delta$ ” definition for  $\lim_{x \rightarrow a} f(x) = L$ , where  $a, L$  are real numbers.  
(b) (10 points) Using the definition in (a), prove that  $\lim_{x \rightarrow 1} 2x + 4 = 6$ .
3. (20 points) Is the function

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

continuous everywhere? Justify your answer with Theorems from class.

4. (20 points) Find the derivative of  $\frac{x+1}{x^2+1} + \sec(x) \tan(x)$ .

5. At time  $t = 0$ , two runners embark on a race. The distance of runner A from the starting line in meters at time  $t$  is  $t\sqrt{t}$ . The distance of runner B from the starting line in meters at time  $t$  is  $\sqrt{t}$ .
- (a) (5 points) Which of the two runners initially takes the lead?
  - (b) (7 points) When does the runner who is initially behind catch up with the other runner?
  - (c) (8 points) At what time  $t > 0$  do the runners have the same velocity?