University of California, Riverside
Department of Mathematics

# Final Exam

Mathematics 8B - First Year of Calculus
Sample 1

Instructions: This exam has a total of 200 points. You have 3 hours. You must show all your work to receive full credit You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

The following formulae may be useful: for any $A, B$, we have

$$
\begin{aligned}
\sin(A+B) &= \sin(A)\cos(B)+\cos(A)\sin(B), \\
\cos(A+B) &= \cos(A)\cos(B)-\sin(A)\sin(B), \\
\tan(A+B) &= \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}, \qquad \tan\left(\frac{A}{2}\right)=\frac{\sin(A)}{1+\cos(A)}=\frac{1-\cos(A)}{\sin(A)}, \\
\sin\left(\frac{A}{2}\right) &= \pm\sqrt{\frac{1-\cos(A)}{2}}, \qquad \cos\left(\frac{A}{2}\right)=\pm\sqrt{\frac{1+\cos(A)}{2}}, \\
\sin(2A) &= 2\sin(A)\cos(A), \qquad \cos(2A)=\cos^2(A)-\sin^2(A), \\
\cos(2A) &= 2\cos^2(A)-1, \qquad \cos(2A)=1-2\sin^2(A), \\
\tan(2A) &= \frac{2A}{1-\tan^2(A)}
\end{aligned}
$$

1. Find the precise values of

   (a) (5 points) $\cos\frac{4\pi}{3}$

   (b) (5 points) $\tan\frac{-7\pi}{12}$

   (c) (5 points) $\tan 15^o$

   (d) (5 points) $\arcsin\frac{-\sqrt{3}}{2}$

2. Determine the following limits.

(a) (10 points) $\lim_{x\to 9} \frac{9-x}{3-\sqrt{x}}$

(b) (10 points)

$$\lim_{x\to 3^-} \frac{x-2}{x-3}$$

3. (10 points) For what value of $a$ is the following function continuous?

$$f(x) = \begin{cases} \frac{1}{x-1} & 1 < x \le 2 \\ \sqrt{2x-a} & 2 < x \end{cases}$$

Justify your answer carefully by arguing with left- and right-hand limits.

4.

(a) (10 points) Find the tangent line to the graph of $f(x) = x^2 + x$ through $(1, 2)$. Do so by computing a limit, not by using a differentiation rule.

(b) (10 points) Find the derivative of $\sin(\sin(\sin x))$. You may use differentiation rules.

5.

(a) (10 points) Find $y'(x)$ by implicit differentiation from $\sqrt{x}+\sqrt{y} = 1$

(b) (10 points) Find $y'(x)$ by implicit differentiation from $\tan(x-y) = \frac{y}{x^2+1}$.

6. (20 points) Each side of a square is increasing at a rate of $6\frac{\text{cm}}{\text{sec}}$. At what rate is the area of the square increasing when the area of the square is 16 $\text{cm}^2$?

7. (20 points) Using both the Intermediate Value Theorem and the Mean Value Theorem, show that the polynomial $x^3 + x - 1$ has exactly one root between 0 and 1. Do not forget to state the theorems!

8. Let $f(x) = \frac{x^2+1}{(x-2)^2}$.

(a) (3 points) Find the domain of $f$.

(b) (3 points) Find the intercepts.

(c) (4 points) Find the vertical and horizontal asymptotes.

(d) (4 points) Find the intervals of increase and decrease.

(e) (8 points) Find the local maximum and minimum values.

(f) (4 points) Find the intervals of upward and downward concavity and points of inflection.

(g) (4 points) Sketch the graph based on the above findings.

9. (20 points) Find a positive number $x$ such that the sum of $x$ and its reciprocal $\frac{1}{x}$ is as small as possible.

10. Find the most general antiderivative of

    (a) (10 points) $f(x) = \frac{3+x^2+x^5}{x^4}$

    (b) (10 points) $g(x) = (4+7x)^5$ .