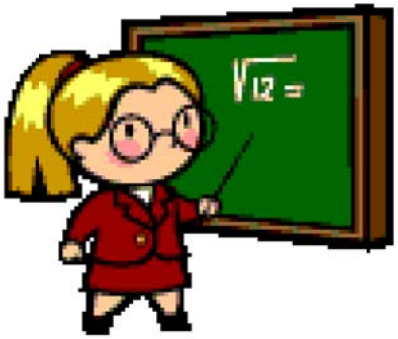
Logic
Day 1 - Introduction



I. **Sentences (2 Types)**

- (has truth value)

1. **Atomic** – simplest, most basic sentence

Ex 1: Today is Thursday.

Ex 2: It is sunny.

2. **Molecular-** made up of 2 or more atomic sentences.

Ex : Today is Thursday and it is sunny

II. Connectives

1. And (Both… and…)

Ex: Math is fun and I can read

2. Or (Either… or…)

Ex: Today is Wednesday or today is Thursday.

3. If…, then… (…implies…)

Ex: If this is October, then Halloween is soon.

4. Not (It is not the case that…)

Ex: The moon is not made up of cheese.
(Atomic: The moon is made up of cheese)

→ When "not" is added to an atomic sentence, the sentence changes to molecular

III. Forms of Molecular Sentences

1. **And:** ( ) and ( )
   Both ( ) and ( )

2. **Or:** ( ) or ( )
   Either ( ) or ( )

1. **If/then:** If ( ), then ( )
   ( ) implies ( )

- **Not:** Usually inside a sentence

Ex: Logic is not difficult
Write "not" in front
Ex: Not (logic is difficult)

Can fill in parenthesis with atomic or molecular sentences!

Ex: Today is sunny and I am a boy or a girl.

IV. Symbolizing Sentences

1. Capital letters- used to denote atomic sentences

Ex: The snow is deep and the weather is cold
P= the snow is deep
Q =the weather is cold

P and Q

2. Not- put “not” in front of capital letters

Ex: Ducks do not have 4 legs
D=ducks do have 4 legs

So, “not D”

V. Symbols for Connectives

1. **And**- Conjunction ∧

2. **Or**- Disjunction ∨

3. **If…, then**- Conditional →

4. **Not**- Negation ~ or ¬

VI. Grouping and Parentheses

1. Sentences can have more than 1 connective
2. One connective is the major or dominant connective

i. Hints for finding the major connective

- commas
- 2- words
    - Both ( ), and ( )
    - Either ( ) or ( )
    - If ( ), then ( )

3. Examples

i. **Both** x = 1 or x = 2, **and** y = 3.

P: x = 1
Q: x = 2
R: y = 3

How do you symbolize it? $(P \lor Q) \land R$

ii. **Either** the game has not started **or** there is not a big crowd.

How many connectives? 3

How do you symbolize it? (~G) V (~B)

VII. Eliminate Some Parentheses

1. When there are no ( )…

   - i. Rule 1: Strongest →

   - ii. Rule 2: Weakest ~

   - iii. Rule 3: ∨ and ∧ have equal strength. When both occur, there must be ( ) to indicate which is the major connective.

Practice Problems

M: It is morning.
F: It is foggy.
C: I am cold.

Symbolize the following:

It is morning and I am cold.

It is not foggy and I am cold.

If it is foggy, then I am cold.

It is morning or it is foggy, and I am cold.

It is morning, or it is foggy and I am cold.

It is neither foggy nor morning.

It is not foggy and it is morning.

Logic- Day 2

PONENDO PONENS

P.P.

Warm- Up

- Symbolize this sentence:

Either the sky is blue or if I am happy then the bluebirds will fly home.

Let A= the sky is blue

B = I am happy

C = the bluebirds will fly home

Warm- Up Answer

$$A \vee (B \rightarrow C)$$

Homework Questions???

Inference and Deduction

Logic is like a game played with symbolized sentences. We begin with a set of statements called “premises.” The object of the game is to use the rules of logic (rules of inference) in such a way that that we are led from premises to the desired conclusion.

Modus Ponendo Ponens (PP)

If you are given a conditional and the antecedent, then you can conclude the consequent is true.

Conditional : A→ B

A is the antecedent

B is the consequent

Modus Ponendo Ponens (PP)

Premise 1 : If it is raining, then the sky is cloudy

Premise 2: It is raining.

What can we infer?

2 Column Proofs (Examples)

1. Prove S

Statement	Reason
1. ~ R	P
2. ~R→ S	P
3. S	PP 1, 2

Premise: what you are given

Q.E.D.

2 Column Proofs (Examples)

2. Prove Q v P

Statement	Reason	
1. ~ P→ ~S	1. P	Given
2. ~P	2. P	
3. ~ S → (Q v P)	3. P	
4. ~S	4. PP 1, 2	
5. Q v P	5. PP 3,4	

2 Column Proofs (Examples)

3. “ *If it is February, then it rains. If it rains, then the ground is wet. It is February;* therefore, the ground is wet.”

F: it is February

R: it rains

G: ground is wet

F → R

R → G

F → G

Prove G.

Example #3, continued

Statement	Reason
1. F → R	1. P
2. R → G	2. P
3. F	3. P
4. R	4. PP 1,3
5. G	5. PP 2, 4

Example #4

x+1 = 2

If x+1= 2, then y+ 1= 2

If y+1 =2, then x= y

Prove x=y

Let A = x+1=2

B = y+1= 2

C= x=y

1. A	P
2. A → B	P
3. B → C	P
4. B	PP 1, 2
5. C	PP 3, 4

#5 #6 & #7

Board time ☺

LOGIC DAY 3

S, A, TT, TP, DN

WARM-UP

Given:

1. S → P
2. P → Q
3. R → S
4. R

Prove Q.

ANSWER

1. S → P	1. P
2. P → Q	2. P
3. R → S	3. P
4. R	4. P
5. S	5. PP 3, 4
6. P	6. PP 1, 5
7. Q	7. PP 2, 6

Homework Questions???

LOGIC PUZZLE:

- Four cards are laid on a table. Each one has a number on one side and a letter on the other side. You see the letters a and b and the numbers 3 and 4. You are told the following rule: that if a card has a vowel on one side then it has an even number on the other side. Which card(s) do you need to flip over to see if the rule is true?

CONJUNCTION & DISJUNCTION

I. Conjunction

$$P \wedge Q$$

Given: Today is Monday ***and*** it is warm.

* Both must be true for the statement to be true.

CONJUNCTION & DISJUNCTION

II. Disjunction

P v Q

- One or both are true
- At least one is true, or possibly both P & Q are true

P v Q

Given false. Thus Q must be true

P v Q

Given false. Thus P must be true

P v Q

Given true. We do not know Q. Could be true or false.

WHAT CAN YOU CONCLUDE? WHAT **RULE** ALLOWS YOU TO CONCLUDE SUCH?

1) $P \rightarrow Q$
P Ponendo Ponens (PP)
Q

2) $P \rightarrow Q$
Q
No Conclusion

3) $P \rightarrow Q$
$\sim P$
No Conclusion

4) $$\begin{array}{l} P \rightarrow Q \\ \sim Q \\ \hline \sim P \end{array}$$

Tolendo Tolens
(TT)

5) $$\begin{array}{l} P \vee Q \\ \sim Q \\ \hline P \end{array}$$

Tolendo Ponens
(TP)

"Ponendo" = accept

"Tolendo" = deny

6) $$\begin{array}{c} \sim\sim P \\ \hline P \end{array}$$

Double Negation
(DN)

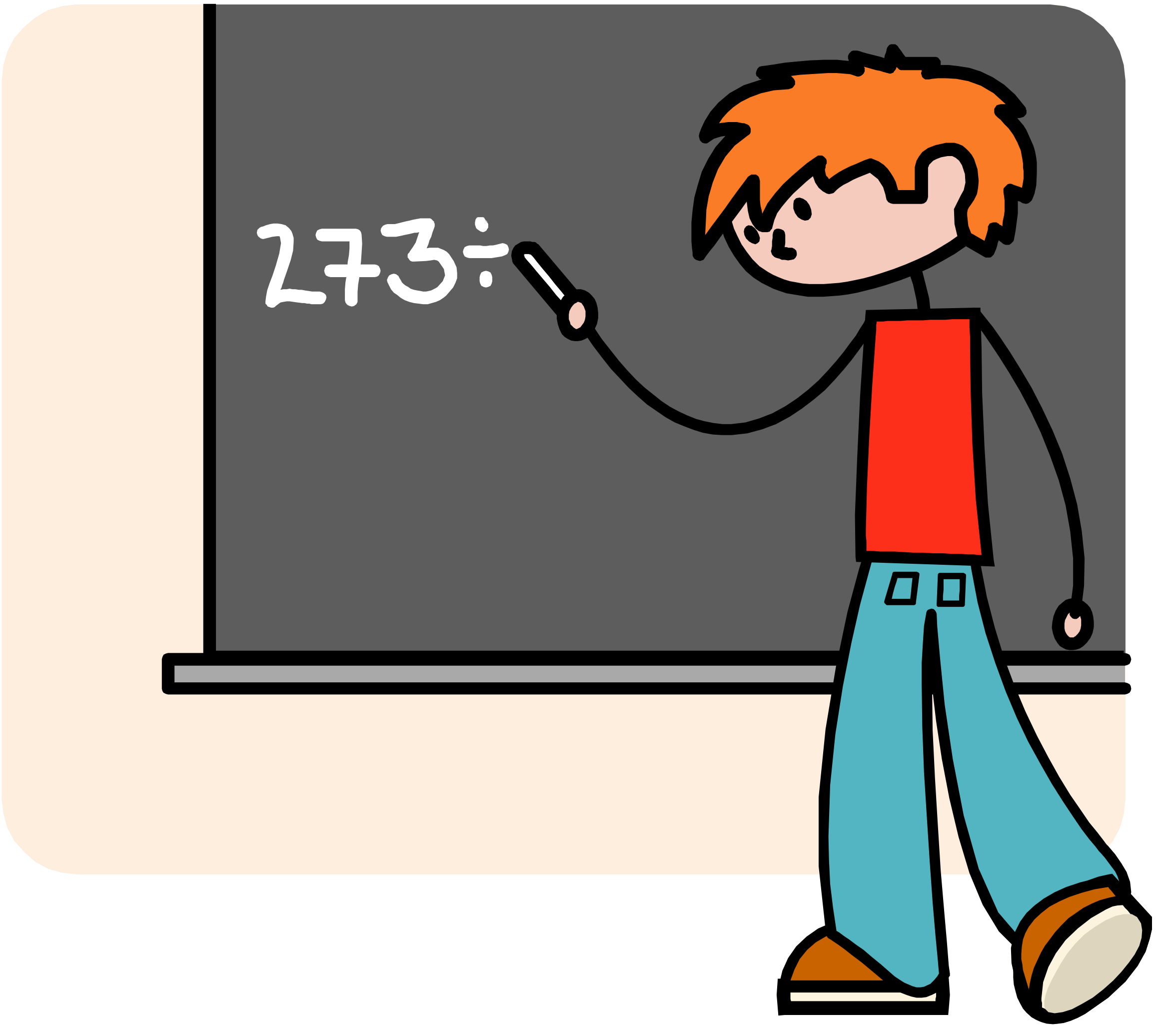
7) $$\begin{array}{l} P \\ Q \\ \hline P \wedge Q \end{array}$$

"Add"junction
Adjunction
(A)

8) $$\begin{array}{l} P \wedge Q \\ \hline P \\ Q \end{array}$$

Simplification
(S)

BOARD TIME!!!



Logic– Day 4

HS, LA

Warm- Up

Prove: A ^ B

1. A ^ C	1. P
2. ~ C V D	2. P
3. E→~ D	3. P
4. (F V B) V E	4. P
5. ~ F	5. P
6. A	6. S1
7. C	7. S1
7.5 ~~C	7.5 DN 7
8. D	8. TP 2,7.5
8.5 ~~D	8.5 DN 8
9. ~E	9. TT 3,8.5
10. F V B	10. TP 4,9
11. B	11. TP 5,10
12. A ^ B	12. A 6,11

Homework Questions???

Today: 2 new rules

Hypothetical Syllogism (HS)

Law of Addition (LA)

Hypothetical Syllogism (HS)

If it is Monday, then I go to school.

P Q

If I go to school, then I go to my classes."

Q R

What can you conclude?

If it is Monday, I go to my classes.

The rule in Symbols:

$$\begin{array}{l} P \rightarrow Q \\ \underline{Q \rightarrow R} \\ P \rightarrow R \end{array}$$

Hypothetical Syllogism looks a lot like…..

<u>The Transitive Property</u>

Ex: Prove A→C

1.A→B	1. P
2. B→ C	2. P
3. A→C	3. HS

Law of Addition (LA)

Recall: **Disjunction**

P V Q…. Given as a premise … What must be true?

at least one of P, Q must be true

What can you conclude?

1. I am a student Premise (given, true)
2. I am a student or I am a lawyer True
3. I am a student or pigs can fly True

Note: only 1 must be true!

Law of Addition (LA) – as long as <u>one</u> part of the disjunction is true, then the whole statement is true.

The rule in symbols: see example below

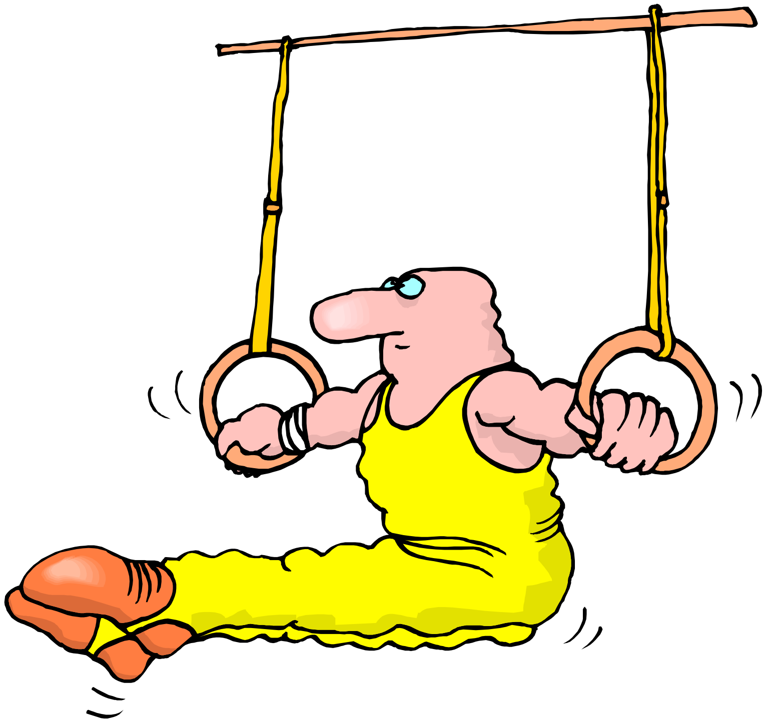
Ex: Prove S V T

1. ~T →A	1. P
2. ~ A	2. P
3. ~~T	3. TT 1,2
4. T	4. DN 3
5. S V T	5. LA 4

No where is S mentioned

Practice time!!!

Practice makes perfect ☺



LOGIC- DAY 5

Five more Rules: DS, DP, CL, LB, DL

The last ones

WARM- UP

Prove: P

1.~ R →(P V S)	1. P
2. (P V S) → (~R → ~T)	2. P
3. S → ~(~R → ~T)	3. P
4. ~R	4. P
5. (P V S)	5. PP 1,4
6. ~ R → ~T	6. PP 2,5
7. ~S	7. TT 3,6
8. P	8. TP 5,7

LAW OF DISJUNCTIVE SYLLOGISM (DS)

Given

Either it is raining or it is sunny	P V Q
If it is raining, then we stay indoors	P→ R
If it is sunny, then we go outside	Q → S

Conclusion: P V Q
P→ R
Q → S
R V S

Either we stay inside or we go outside

(DS)

LAW OF DISJUNCTIVE SIMPLIFICATION (DP)

Given:

We will have a test or we will have a test.

T V T

Conclusion: $\underline{\text{T V T}}$

T (DP)

When is it ever used?

P V Q
P→ R
$\underline{\text{Q → R}}$
$\underline{\text{R V R}}$ DS
R DP

COMMUTATIVE LAW (CL)

2 + 3 = 3 + 2

A V B = B V A

A ^ B = B ^ A

LAW OF BICONDITIONAL (LB)

I get an A if and only if my grade is 90% or above.

$A \leftrightarrow B$ means $A \rightarrow B$

$B \rightarrow A$

Example:

1. $\underline{A \leftrightarrow B}$
2. $A \rightarrow B$ LB 1
3. $B \rightarrow A$ LB 1

1. $C \rightarrow D$
2. $\underline{D \rightarrow C}$
3. $C \leftrightarrow D$ LB 1,2

DE MORGAN'S LAW (DL)

It is not cold or it is not hot.
It is not both hot and cold

The same
sentence

~ C V ~ H
~ (C ^ H)

The same logic

Laws

1. Change ^ to V, or V to ^
2. Negate each member of the conj/disjunction
3. Negate the whole formula

DE MORGAN'S LAW: EXAMPLES

1. P ^ Q = ~(~ P V ~Q)
2. ~ P V ~Q = ~(P ^ Q)
3. ~(P V Q) = ~P ^ ~Q
4. ~(~P ^ Q) = P V ~Q

PRACTICE TIME!!!

Let's do #1 and #2 together!

1) Prove: P if and only if Q

1. $\sim (P \rightarrow Q) \rightarrow R$	1. P	$\leftrightarrow$
2. $\sim R$	2. P	
3. $Q \rightarrow S$	3. P	
4. $S \rightarrow P$	4. P	
5. $Q \rightarrow P$	5. HS 3,4	
6. $P \rightarrow Q$	6. TT 1,2	
7. $P \leftrightarrow Q$	7. LB 5,6	(Biconditional)

MORE PRACTICE

2) Prove: ~ A

1. ~ B → C	1. P
2. ~ D → C	2. P
3. A → ~ C	3. P
4. ~ (D ^ B)	4. P
5. ~ D v ~ B	5. DL 4
6. C v C	6. DS 5,1,2
7.C	7. DP 6
8.~A	8. TT 3,7

YOUR TURN!

Try #3 and #4 ☺

Logic
Day 6 - Truth Validity

Warm-up

Prove $S \wedge T$

1. $\sim(P \vee \sim R)$
2. $Q \vee P$
3. $R \rightarrow S$
4. $(Q \wedge S) \rightarrow (T \wedge S)$

Today: Truth Validity

Every sentence is either <u>True</u> or <u>False</u>.

1. Conjunction- “AND” P ^ Q

Look at all possible cases!

P	Q	P ^ Q
T	T	T
T	F	F
F	T	F
F	F	F

Called a truth table

2 letters….4 cases

3 letters…8 cases

4 letters…16 cases

2. Disjunction- “or” P V Q

P	Q	P V Q
T	T	
T	F	
F	T	
F	F	

3. Negation- “not” ~

P	~P
T	F
F	T

4. **Conditionals**

Conditionals are different from English. The subject matter of the antecedent does **<u>not</u>** need to relate to the subject matter of the consequent.

*There is no sense of causality; we do not care if the P causes the Q

Ex: If today is a weekday, then 3+3=6.
T T True

Ex: If today is Saturday, then 3+3=6.
F T True

The conditions of truth and falsity for conditionals is the hardest of all the compound statements to understand. Perhaps it is best to remember that a conditional is a hypothetical statement - such that it is asserting that IF…some state of affairs were true, THEN…. some other state of affairs would be true. The tricky thing about this is that if the antacedent of the conditonal is false, then just about anything follows from it. So, for example, the statement, "if today is Friday then tomorrow is Thursday would be true today (Wednesday) but false the day after tomorrow.

4. Conditional- “if, then”

P: I live in palo alto
~P: I do not live in palo alto

Q: I live in California
~Q: I do not live in CA

P	Q	$P \rightarrow Q$
T	T	
T	F	
F	T	
F	F	

Biconditional- “ If and only if ”

Biconditional is true iff both are true or both are false

P	Q	$P \rightarrow Q$	Q	P	$Q \rightarrow P$	$P \leftrightarrow Q$
T	T		T	T		
T	F		F	T		
F	T		T	F		
F	F		F	F		

Diagramming Sentences- Determine whether the sentence is true or false.

*Work from the inside out!

** Evaluate from least powerful to most powerful

1.) Given

P-true
Q- false

R-true

$(P \vee Q) \wedge R$

2.) Given

P-true
Q-true
R-false
S-false

$(P \vee Q \rightarrow P) \wedge (R \vee S)$

3.) Given

A- true
B-false
C-false

$[(A \vee B) \wedge \sim A] \rightarrow \sim (C \rightarrow A)$

VALID Arguments vs. INVALID Arguments

Definition: Valid Argument -- premises are true, conclusion is true

$P \longrightarrow Q$
P
$\overline{Q}$

Ex:

P	Q	**P** **Q**	**Q**

To show that an argument is VALID…

1. List atomic sentences
2. List premises
3. List conclusion
4. Find the case where all premises are true and your conclusion is true

VALID Arguments vs. INVALID Arguments

Definition: INValid Argument -- premises are true, conclusion is false

$P \rightarrow Q$
Q

P

Ex:

P	**Q**	**P Q**	**P**

To show that an argument is INVALID…

1. Symbolize premises & conclusion
2. Assign Truth Table to atomic parts (you pick)
3. Find **1** case where premise = true & conclusion = false

Practice Problems: Show that the argument is INVALID

Prove: S

3 letters -- 8 cases

1. $T \wedge S \leftrightarrow R$
2. ~R
3. T

R	S	T	~R	$T \wedge S \leftrightarrow R$	S

Practice Problems: Show that the argument is INVALID

Prove: S

1. $T \wedge S \leftrightarrow R$
2. $\sim R$
3. T

3 letters -- 8 cases
****Force one case**
Truth Assignment

R	S	T	~R	$T \wedge S \leftrightarrow R$	S

Practice Problems: Show that the argument is INVALID

If it is sunny, then Lucy will walk her dog. It is sunny. Either Lucy will walk her dog or the dog will not learn to fetch a stick. Therefore, the dog will learn to fetch a stick

Truth and Validity (Day 6 – extra notes)

So far in this unit, we have used rules of logic (PP, TP, TT, etc) to derive (prove) conclusions from a given set of premises. We have also tried to determine whether a conclusion logically follows from a given set of premises (is the argument valid or invalid?).

Example:

$$\begin{array}{l} Q \\ \underline{P \rightarrow Q} \\ P \end{array}$$

We showed this argument to be invalid by using a counter-example.

We live in California. (premise)
If we live in Palo Alto, then we live in California. (premise)

But we don't necessarily live in Palo Alto

In this example we showed that a set of true premises led to a false conclusion. In a valid argument, true premises always lead to true conclusions. In many cases, we were able to show a particular argument was valid because we could derive the conclusion using rules of logic.

In this lesson we are going to develop a more rigorous (but easy to use) method for determining the validity or invalidity of an argument.

Truth Value:

Every sentence has a truth value – true or false. This includes atomic and molecular sentence. The truth value of a molecular sentence depends only on the truth values of its atomic sentences and which connectives are used.

Conjunction: A conjunction $P \wedge Q$ is true if and only if both P and Q are true. In all other cases, $P \wedge Q$ is false. We can use a truth table to illustrate this.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction: A disjunction $P \vee Q$ is true if P is true or Q is true or both are true. The only case where it is false is when both P and Q are false.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation: The negation of a true sentence is false and the negation of a false sentence is true.

P	$\sim P$
T	F
F	T

Conditional: In logic, the truth or falsity of a conditional $P \rightarrow Q$ depends only on the truth values of the antecedent (P) and the consequent (Q). The subject matter of the antecedent and consequent need not be related. There does not need to be a causal effect between the antecedent and the consequent. The four possible cases are illustrated in a truth table

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

The first two cases involve a true antecedent. If a true consequent follows from a true antecedent, we say the conditional is true. If a false consequent follows from a true antecedent, we say the conditional is false. These two cases are consistent with our everyday use of conditionals.

If the antecedent is false, the situation is more complicated. Since a conditional statement only implies an outcome when the antecedent is true, there is no test for the conditional when the antecedent is false. A conditional only says what will happen if the antecedent is true. Since a conditional cannot be tested without a true antecedent, the conditional is taken as true whenever the antecedent is false. When the antecedent is false, the conditional is true, regardless of the truth value of the consequent.

To summarize, in the four cases illustrated above, the only time a conditional is false is when the antecedent is true and the consequent is false.

Biconditional: The biconditional $P \leftrightarrow Q$ is the same as $P \rightarrow Q$ and $Q \rightarrow P$. For the biconditional to be true, both conditionals must be true. From the discussion above about conditionals, it follows that a biconditional is true only if the two parts (P and Q) are both true or both false.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Value of Sentences:

The truth value of a sentence can be worked out if we know the truth value of its atomic parts.

Example:

```
(P ∨ Q → R) ∧ (R ∨ S)
 F   T   T     T   F
 \   /   /      \  /
   T    /        T
    \  /        /
     T         /
       \      /
          T
```

Given: P – false
Q – true
R – true
S – false

Validity of an Argument:

In a valid argument, true premises must always lead to true conclusions. To show that an argument is valid, we have used rules of logic to show that a conclusion can be derived from a set of premises. This is not always easy. Using a truth table, we can systematically show whether true conclusions always follow from true premises (valid argument) or if there are one or more cases where a false conclusion follows from true premises (invalid argument).

Consider the example:

P → Q
~P
~Q

We know this argument to be invalid, but let's prove it using a truth table.

P	Q	P → Q	~P	~Q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

The third row shows a case where both premises (P → Q and ~P) are true, but the conclusion (~Q) is false. This is proof that the argument is invalid.

Sometimes it is possible to skip the process of filling in the entire truth table and to instead find one set of truth values that make the premises true and the conclusion false. This shortcut is called *truth assignment*.

Reference: First Course in Mathematical Logic, Suppes and Hill (pages 110-129)

LOGIC- Day 7

Conditional Proof (CP)

Pretty tricky!

WARM- UP

Find the truth-value of this sentence

Given: True- P, Q   False- B

$$[(P \wedge Q) \rightarrow \sim B] \rightarrow [P \rightarrow (Q \rightarrow B)]$$

From Hwwk : Section B, #3

PROVE: ~Q

1. T → Q
2. ~T V R
3. ~R

Let’s do it the long way….

Truth Table

Q	R	T	T → Q	~T V R	~R	~Q
T	T	T	T	T	F	F
T	T	F	T	T	F	F
T	F	T	T	F	T	F
T	F	F	T	T	T	F ←
F	T	T	F	T	F	T
F	T	F	T	T	F	T
F	F	T	F	F	T	T
F	F	F	T	T	T	T

PROVE: T

1. ~(P V Q)
2. P V R
3. T → R

Let’s do it the Short way….

<u>Truth Assignment</u>

P	Q	R	T	~(P V Q)	P V R	T → R	T
F	F	T	F	(DL) ~P ^ ~Q T T **T**	F T **T**	F T **T**	**F**

Yes or No???

1.) A valid argument has true premises and a false conclusion?

NO - A valid argument has true premises and a true conclusion

2.) The only way to show that an argument is valid, is by using truth table?

NO - We can also do a two column proof

3.) De Morgan's law can be applied to part of this statement?

$$\sim A \vee B \longrightarrow C$$

YES- the (~A V B) part…..

Today: Conditional Proof

* We have only been looking at direct proofs so far*

<u>Given</u>

If I live in PA, the I live in CA	$P \to C$
If I live in Boston, then I do not live in CA	$\underline{B \to \sim C}$
Therefore, if I live in Boston, then I do not live in PA.	$B \to \sim P$

---

1. This makes sense but we do not have sufficient rules to prove true.

2. Check if valid (True premise & True Conclusion)

    -Truth Table

P	C	B	P → C	B → ~C	B → ~P	
T	T	T	T	F	F	
T	T	F	T	T	T	←
T	F	T	F	T	F	
T	F	F	F	T	T	
F	T	T	T	F	T	
F	T	F	T	T	T	←
F	F	T	T	T	T	←
F	F	F	T	T	T	←

This argument is VALID!!!!

3.) Therefore we have to introduce a new rule

Law of Conditional Proof (CP)-

If we are able to derive a sentence S from a sentence R and a set of premises, then we may derive R → S from the set of premises alone

Steps to use CP:

If desired conclusion is a conditional

1) Add antecedent as premise

2) Indent right

3) Derive consequent from the original and new premises.

Back to our example:

$P \rightarrow C$

$B \rightarrow \sim C$

$B \rightarrow \sim P$

If I assume the “if” is true, can I get the ‘then’????

Prove: $B \rightarrow \sim P$

1. $P \rightarrow C$	1. P (original)
2. $B \rightarrow \sim C$	2. P (original)
3. B (the antecedent)	3. P (new) ← Stuck :/
4. $\sim C$	4. PP 2,3
5. $\sim P$	5. TT 1,4
6. $B \rightarrow \sim P$	6. CP 3-5

When I assumed B was true, I was able to conclude ~P

Tab left b/c we derived from the original premises alone
b/c of law of CP

You Try one!!!

Prove: D → C	
1. A →(B → C)	1. P (original)
2. ~ D V A	2. P (original)
3. B	3. P (original)
4. D	4. P (new)
5. A	5. TP 2,4
6. B →C	6. PP 1,5
7. C	7. PP 3,6
8. D→C	8. CP 4-7

A CHALLENGE!!!!-- Work with your group :)

Prove: P → (~Q → R)

1. S ^ (~P V M)
2. M → Q V R

1. P (original)
2. P (original)

Geometry/Algebra 2

1) Logic Day 8
   - Warm-up
   - Notes
   - Homework

Prove $A \longrightarrow B$

1. $(A \vee C) \longrightarrow D$	1. P
2. $E \longrightarrow (\sim D \wedge \sim F)$	2. P
3. $E \vee B$	3. P

<u>LOGICALLY IMPOSSIBLE</u> Sentence

$R \wedge \sim R$?!?!?!??! Contradiction

<u>Definition: Contradiction</u>

2 sentences are contradictory if one is a negation of the other. A <u>contradiction</u> is the conjunction of a sentence and its negation. It is always <u>false</u>

Ex: Derive a contradiction (We’re not proving anything here)

1. A	P
2. $\sim C$	P
3. $B \to C$	P
4. $A \to B$	P

We arrived at a CONTRADICTION!!! (Logically False)

At least one of the premises must be false because you can't get a false conclusion from true premises using valid logic. Thus, if the conclusion is logically false, at least one of the premises is false also.

Types of Proofs:

1. Direct

2. Conditional

3. Indirect (Proof by contradiction) -- NEW

Steps for Indirect Proof:

1. introduce negation of desired conclusion (new premise)

2. derive a contradiction

3. state the desired conclusion

Prove ~ D

1. A V ~B
2. D $\longrightarrow$ ~A
3. B

Rule for Indirect Proof

(RAA) reductio ad absurdum - to reduce to the absurd, a contradiction

Note: an indentation or subordinate proof can be ended only when using CP or RAA

Prove: ~D

1. D → W
2. A V ~W
3. ~(D V A)

Prove: ~B

1. J ^ B ⟶ S
2. ~S V T
3. ~T
4. J

Logic- Day 9

Validity & Tautology

Warm- Up

Prove $\sim (W \wedge R)$

1. $W \rightarrow T$	1. P
2. $T \rightarrow S$	2. P
3. $R \rightarrow \sim S$	3. P
4. $W \rightarrow S$	4. HS 1,2
5. $W \wedge R$	5. P (new)
6. W	6. S5
7. R	7. S5
8. $\sim S$	8. PP 3,7
9. S	9. PP 4,7
10. $S \wedge \sim S$	10. A 8,9
11 $(W \wedge R)$	11. RAA 5,10

Logic Day 9

Why do we end up with a contradiction?

One of the premises is false. It cannot exist with other premises; not all true at the same time!

Definition: Inconsistent Premises

- 2 or more sentences that cannot all be true together. They do not get along :) You WILL end up with a contradiction

Consistent Premises (all are true)

2 or more sentences that can be true together (at the same time)

Prove: R

1. $\sim (P \land R) \rightarrow Q$
2. $\sim Q$
3. $P \land R$
4. R

1. P
2. P
3. P
4. S 3

Conclusion is True.

Thus, premises are True….

Therefore, …..

Consistent Premises!!!

Consistent Premises

To show that a set of premises is CONSISTENT…

find one case, using a truth table where all premises cane be true together.

Tautology

Definition- a final molecular sentence that is always TRUE!!!

Ex: (P V P)→ P

P	P V P	P V Q → P
T	T	T
F	F	T

* Break up the statement into parts

All true,

∴ tautology

Tautology

Definition- a final molecular sentence that is always TRUE!!!

Ex: P V Q → P

P	Q	P V Q	P V Q → P
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

* Break up the statement into parts

Why is this important? If a conclusion is a tautology, then it is valid!!!

Not tautology

You Try!!!

Is P V ~ (P ∧ Q) a tautology?!?!

Yes!!!!

Tautological Implication

Definition: a tautology in the form of a conditional.
Used to show if an argument is valid or invalid

How to construct a Tautological Implication (TI) : a conditional

1. Antecedent - conjunction of premises
2. Consequent- desired conclusion (what you are trying to prove)

Prove: $R \wedge S$

1. P	1. P
2. $P \longrightarrow Q$	2. P
3. $\sim Q \vee (R \wedge S)$	3. P

⇨ $P \wedge (P \longrightarrow Q) \wedge (\sim Q \vee (R \wedge S)) \longrightarrow R \wedge S$

You Try!! TI

Write the following rules as TI's

PP

TT

HS

How to deconstruct a TI

Work Backwards!!!

Directions: Construct an argument

$P \wedge (P \rightarrow Q) \longrightarrow \sim(P \wedge \sim Q)$

TI's

Tautological Implications can be used to…show whether an argument is valid. We can make any argument into a TI

Steps:

1. Change argument to TI
2. Use truth table to determine if it is a tautology
3. If tautology, then the argument is valid.

   If not tautology, then the argument is invalid

Check if PP is valid

$(P \longrightarrow Q) \land P \longrightarrow Q$

P	Q	$P \longrightarrow Q$	$(P \longrightarrow Q) \land P$	$(P \longrightarrow Q) \land P \longrightarrow Q$

You try- show the argument is INVALID

$(P \rightarrow Q) \wedge \sim P \longrightarrow \sim Q$

P	Q	$P \rightarrow Q$	$\sim P$	$\sim Q$	$(P \rightarrow Q) \wedge \sim P$	$(P \rightarrow Q) \wedge \sim P \rightarrow \sim Q$

One last thing….

Logical Equivalence - 2 arguments whose truth tables are identical

Ex: P V ~Q vs ~(~P ∧ Q)

P	Q	P V ~Q
T	T	T
T	F	T
F	T	F
F	F	T

P	Q	~P ∧ Q	~(~P ∧ Q)
T	T	F	T
T	F	F	T
F	T	T	F
F	F	F	T

LOGIC
HW 1

A) 1. $(P \wedge Q) \rightarrow R$
2. $(P \vee Q) \rightarrow R$
3. $(P \wedge \sim Q) \rightarrow \sim R$
4. $R \rightarrow (P \vee \sim R)$
5. $R \wedge (P \wedge Q)$
6. $\sim R \rightarrow \sim P$
7. $P \vee Q$
8. $(\sim Q \vee P) \rightarrow R$
9. $R \wedge (P \vee Q)$
10. $Q \wedge (P \rightarrow R)$
11. $\sim (P \wedge Q)$
12. $(\sim P \wedge Q) \rightarrow \sim R$

B) 1. $P \rightarrow Q$
2. $P \vee Q$
3. $(P \vee Q) \rightarrow \sim R$
4. $\sim P \vee \sim Q$
5. $(P \wedge Q) \vee (R \wedge S)$
6. $\neg (P \wedge Q)$
7. $\sim (P \vee Q)$
★ 8. $\sim P \rightarrow (\sim Q \wedge R)$
★ 9. $\sim (P \rightarrow Q)$
10. $\sim (P \wedge \sim P)$
11. $P \wedge (Q \vee R)$
12. $(P \wedge Q) \vee R$
13. $P \wedge (Q \rightarrow \sim R)$

C) 1. c, h
2. b, e
3. a
4. i
5. f
6. d
7. g
8. j

★ D) ☆ 1. $P \vee Q$
☆ 2. $R \wedge S \rightarrow T$
☆ 3. $(A \vee B) \rightarrow \sim C$

Logic HW 2 Answers day 2

2.2, Exercise 3, pages 50–52

A. 1. Prove: $\neg T$

(1)	$R \to \neg T$	P
(2)	$S \to R$	P
(3)	S	P
(4)	R	PP 2, 3
(5)	$\neg T$	PP 1, 4

2. Prove: G

(1)	$\neg H \to \neg J$	P
(2)	$\neg H$	P
(3)	$\neg J \to G$	P
(4)	$\neg J$	PP 1, 2
(5)	G	PP 3, 4

3. Prove: C

(1)	$A \to B \,\&\, D$	P
(2)	$B \,\&\, D \to C$	P
(3)	A	P
(4)	$B \,\&\, D$	PP 1, 3
(5)	C	PP 2, 4

4. Prove: $M \vee N$

(1)	$\neg J \to M \vee N$	P
(2)	$F \vee G \to \neg J$	P
(3)	$F \vee G$	P
(4)	$\neg J$	PP 2, 3
(5)	$M \vee N$	PP 1, 4

5. Prove: $\neg S$

(1)	T	P
(2)	$T \to \neg Q$	P
(3)	$\neg Q \to \neg S$	P
(4)	$\neg Q$	PP 2, 1
(5)	$\neg S$	PP 3, 4

C. 1. Prove: $\neg N$

(1)	$R \to \neg S$	P
(2)	R	P
(3)	$\neg S \to Q$	P
(4)	$Q \to \neg N$	P
(5)	$\neg S$	PP 1, 2
(6)	Q	PP 3, 5
(7)	$\neg N$	PP 4, 6

2. Prove: $R \vee S$

(1)	$C \vee D$	P
(2)	$C \vee D \to \neg F$	P
(3)	$\neg F \to A \,\&\, \neg B$	P
(4)	$A \,\&\, \neg B \to R \vee S$	P
(5)	$\neg F$	PP 2, 1
(6)	$A \,\&\, \neg B$	PP 3, 5
(7)	$R \vee S$	PP 4, 6

B. *1. Let P = '2 is greater than 1'
Q = '3 is greater than 1'
R = '3 is greater than 0'

Prove: R

(1)	$P \to Q$	P
(2)	$Q \to R$	P
(3)	P	P
(4)	Q	PP 1, 3
(5)	R	PP 2, 4

2. Prove: $x = y$

(1)	$x+1=2$	P
(2)	$x+1=2 \to y+1=2$	P
(3)	$y+1=2 \to x=y$	P
(4)	$y+1=2$	PP 2, 1
(5)	$x=y$	PP 3, 4

3. Prove: $x+2=y+2$

(1)	$x+0=y \to x=y$	P
(2)	$x+0=y$	P
(3)	$x=y \to x+2=y+2$	P
(4)	$x=y$	PP 1, 2
(5)	$x+2=y+2$	PP 3, 4

4. Prove: $x>10$

(1)	$(x>y \;\&\; y>z) \to (x>z)$	P
(2)	$x>y \;\&\; y>z$	P
(3)	$(x>z) \to (x>10)$	P
(4)	$x>z$	PP 1, 2
(5)	$x>10$	PP 3, 4

5. Prove: $z=x$

(1)	$x=y \;\&\; y=z \to x=z$	P
(2)	$x=z \to z=x$	P
(3)	$x=y \;\&\; y=z$	P
(4)	$x=z$	PP 1, 3
(5)	$z=x$	PP 2, 4

6. Let R = 'The moist air rises'
L = 'It will cool'
M = 'Clouds will form'

Prove: M

(1)	$R \to L$	P
(2)	$L \to M$	P
(3)	R	P
(4)	L	PP 1, 3
(5)	M	PP 2, 4

B = 'The mother's brother is head of the family'
D = 'The father does impose discipline'

roy $\neg D$

	$\rightarrow B$	P
	$\rightarrow \neg D$	P
3)	M	P
4)	B	PP 1, 3
5)	$\neg D$	PP 2, 4

S = 'It is a sedimentary rock'
G = 'This rock is granite'

2. Prove: I

(1)	$I \lor S$	P
(2)	G	P
(3)	$G \rightarrow \neg S$	P
(4)	$\neg S$	PP 3, 2
(5)	I	TP 1, 4

J: Jack is taller than Bob
M = 'Mary is shorter than Jean'
S = 'Jack and Bill are the same heig

3. Prove: $\neg S$

(1)	$J \rightarrow M$	P
(2)	$\neg M$	P
(3)	$S \rightarrow J$	P
(4)	$\neg J$	TT 1, 2
(5)	$\neg S$	TT 3, 4

↑A
↙B

HW #3

1. Prove: Q

(1)	$S \rightarrow (P \lor Q)$	P
(2)	S	P
(3)	$\neg P$	P
(4)	$P \lor Q$	PP 1, 2
(5)	Q	TP 4, 3

2. Prove: R

(1)	$S \rightarrow \neg T$	P
(2)	T	P
(3)	$\neg S \rightarrow R$	P
(4)	$\neg S$	TT 1, 2
(5)	R	PP 3, 4

3 ve: $S \,\&\, T$

(1)	$P \,\&\, R$	P
(2)	$P \rightarrow S$	P
(3)	$R \rightarrow T$	P
(4)	P	S 1
(5)	S	PP 2, 4
(6)	R	S 1
(7)	T	PP 3, 6
(8)	$S \,\&\, T$	A 5, 7

4 Prove: $\neg S$

(1)	$T \rightarrow R$	P
(2)	$R \rightarrow \neg S$	P
(3)	T	P
(4)	R	PP 1, 3
(5)	$\neg S$	PP 2, 4

5 Prove: T

(1)	$P \rightarrow S$	P
(2)	$\neg S$	P
(3)	$\neg P \rightarrow T$	P
(4)	$\neg P$	TT 1, 2
(5)	T	PP 3, 4

6 ve: $S \,\&\, T$

	$P \rightarrow S$	P
(2)	$P \rightarrow T$	P
(3)	P	P
(4)	S	PP 1, 3
(5)	T	PP 2, 3
(6)	$S \,\&\, T$	A 4, 5

7. Prove: S

(1)	$P \lor Q$	P
(2)	$\neg Q$	P
(3)	$P \rightarrow S$	P
(4)	P	TP 1, 2
(5)	S	PP 3, 4

8. Prove: S

(1)	$T \rightarrow R$	P
(2)	$\neg R$	P
(3)	$T \lor S$	P
(4)	$\neg T$	TT 1, 2
(5)	S	TP 3, 4

9. Prove: $\neg T$

(1)	$P \leftrightarrow S$		P
(2)	$P \,\&\, Q$		P
(3)	$(S \,\&\, R) \rightarrow \neg T$		P
(4)	$Q \rightarrow R$		P
(5)	P	(5) P S,2	S 2
(6)	S	OR * (6) Q S,2	PP 1, 5
(7)	Q	(7) S PP1,5	S 2
(8)	R		PP 4, 7
(9)	$S \,\&\, R$		A 6, 8
(10)	$\neg T$		PP 3, 9

10. Prove: $\neg R$

(1)	$S \lor \neg R$	P
(2)	$T \rightarrow \neg S$	P
(3)	T	P
(4)	$\neg S$	PP 2, 3
(5)	$\neg R$	TP 1, 4

11. Prove: S

(1)	$P \rightarrow Q \,\&\, R$	P
(2)	P	P
(3)	$T \rightarrow \neg Q$	P
(4)	$T \lor S$	P
(5)	$Q \,\&\, R$	PP 1, 2
(6)	Q	S 5
(7)	$\neg T$	TT 3, 6
(8)	S	TP 4, 7

* If they are not in the same order as yours that's O.K. For example, #9 Simplification could be in steps 5+6 instead of 5,

12. Prove: $\neg Q$

(1)	$T \lor \neg S$	P
(2)	S	P
(3)	$Q \rightarrow \neg T$	P
(4)	T	TP 1, 2
(5)	$\neg Q$	TT 3, 4

13. Prove: $Q \lor R$

(1)	$S \rightarrow \neg T$	P
(2)	T	P
(3)	$\neg S \rightarrow (Q \lor R)$	P
(4)	$\neg S$	TT 1, 2
(5)	$Q \lor R$	PP 3, 4

14. Prove: S

(1)	$\neg T \lor R$	P
(2)	T	P
(3)	$\neg S \rightarrow \neg R$	P
(4)	R	TP 1, 2
(5)	S	TT 3, 4

15. Prove: $\neg R$

(1)	$Q \,\&\, T$	P
(2)	$Q \rightarrow \neg R$	P
(3)	$T \rightarrow \neg R$	P
(4)	Q	S 1
(5)	$\neg R$	PP 2, 4

↗ if you have more steps,

Day 4
Logic HW

A) ① Prove: $y \nless 4 \lor x > 2$

1. $x > 3 \lor y \nless 4$	P
2. $x > 3 \rightarrow x > y$	P
3. $x \ngtr y$	P
4. $x \ngtr 3$	TT 2,3
5. $y \nless 4$	TP 1,4
6. $y \nless 4 \lor x > 2$	LA 5 [illegible]

③ Prove: $x = 5 \land x \neq 4$

1. $\overset{R}{x = 2} \overset{\rightarrow}{\rightarrow} \overset{S}{x < 3}$	P
2. $x \neq 4 \land x \nless 3$	P
3. $(x \neq 2 \lor x > 4) \rightarrow x = 5$	P
4. $x \nless 3$	S 2
5. $x \neq 2$	TT 1,4
6. $x \neq 2 \lor x > 4$	LA 5
7. $x = 5$	PP 3,6
8. $x \neq 4$	S 2
9. $x = 5 \land x \neq 4$	A 7,8

④ Prove $x = 3$

1. $x - 2 = 1 \land 2 - x \neq 1$	P
2. $x = 1 \rightarrow 2 - x = 1$	P
3. $x = 1 \lor x + 2 = 5$	P
4. $(x + 2 = 5 \lor x - 2 = 1) \rightarrow x = 3$	P
5. $2 - x \neq 1$	S 1
6. $x \neq 1$	TT 5,2
7. $x + 2 = 5$	TP 6,3
8. $x + 2 = 5 \lor x - 2 = 1$	LA 7
9. $x = 3$	PP 4,8

② Prove: $x \neq 3 \lor x > 2$

1. $x + 2 \neq 5 \lor 2x = 6$	P
2. $x + 2 \neq 5 \rightarrow x \neq 3$	P
3. $2x - 2 = 8 \rightarrow 2x \neq 6$	P
4. $x + 3 = 8 \land 2x - 2 = 8$	P
5. $2x - 2 = 8$	S4
6. $2x \neq 6$	PP 3,5
7. $x + 2 \neq 5$	TP 6,1
8. $x \neq 3$	PP 2,7
9. $x \neq 3 \lor x > 2$	LA 8

⑤ Prove: $y < 3 \lor x > 5$

1. $y < 4 \land x = y + 3$	P
2. $x = y + 3 \rightarrow x > 2$	P
3. $y \ngtr 2 \rightarrow x \ngtr 2$	P
4. $(y > 2 \lor y = 3) \rightarrow x > 5$	P
5. $x = y + 3$	S1
6. $x > 2$	PP 5,2
7. $y > 2$	TT 6,3
8. $y > 2 \lor y = 3$	LA 7
9. $x > 5$	PP 8,4
10. $y < 3 \lor x > 5$	LA 9

(B1) 4. $Q \rightarrow R$	S1
5. $\sim T$	PP 4,3
(B2) 4. $\sim Q$	TT 1,3
5. P	TT 4,2
(B3) 5. $\sim R$	TT 3,4
6. S	PP 1,5
7. $P \land Q$	PP 6,2
8. Q	S 7

2 ROUTES
↙ can use HS

C1) 3. $(2+2)+2=6 \rightarrow 3+3=6$ HS:1,2.

C2) 3. $5X-4=3X+4 \rightarrow 2X=8$ HS 1,3

4. $5X-4=3X+4 \rightarrow X=4$ HS 3,2

C3) 4. $x>y \rightarrow z<7$ HS 1,3

5. $z>6$ TT 4,2

6. $z>6 \vee z<y$ LA 5

C4) 5 $x>5 \rightarrow y<x$ HS 1,4

6. $y=5$ PP 2,5

7 $x=6$ TP 6,3

8 $x=6 \vee x>6$ LA 7

C5) 5. $x \neq y \rightarrow x \neq 4$ HS, 1,2

6 $x \neq y$ TT 5,3

7 $x>y \vee x<y$ PP 1,4

8 $x>y$ TP 6,7

C6) 5. $x=3 \rightarrow x<z$ HS 1,4

6 $x \nless z$ PP 5,3

7 $x \ngtr y$ TT 6,4

8 $x \neq 3$ TT 7,1

9. $z=5$ PP 8,2

10 $\sim(z \neq 5)$ DN 9

11 $\sim(z \neq 5) \vee z>5$ LA 10

Day 5
Logic HW

Ⓐ 1) $\sim P \vee \sim Q$
2) $\sim(R \wedge T)$
3) $R \vee \sim S$
④ $\sim(G \wedge H)$
5) $\sim(\sim S \wedge T)$
6) $\sim(P \vee Q)$
7) $\sim(\sim P \vee Q)$
8) $\sim C \wedge \sim D$
9) $P \vee Q$
10) $\sim(S \wedge \sim T)$
11) $\sim(\sim R \vee \sim S)$
⑫ $(A \vee B)$

Ⓑ ① Prove $\sim S$

1.	$\sim(P \wedge Q)$	P
2.	$\sim Q \rightarrow T$	P
3.	$\sim P \rightarrow T$	P
4.	$S \rightarrow \sim T$	P
5.	$\sim P \vee \sim Q$	DL 1
6.	$T \vee T$	DS 5, 2, 3
7.	T	DP 6
8.	$\sim S$	TT 4, 7

(B2) Prove: $\sim(A \vee B)$

1.	$C \wedge \sim D$	P
2.	$C \rightarrow \sim A$	P
3.	$D \vee \sim B$	P
4.	C	S 1
5.	$\sim A$	PP 2, 4
6.	$\sim D$	S 1
7.	$\sim B$	TP 3, 6
8.	$\sim A \wedge \sim B$	A 5, 7
9.	$\sim(A \vee B)$	DL 8

(B3) Prove: $R \wedge Q$

1.	$\sim S \rightarrow \sim(P \vee \sim T)$	P
2.	$T \rightarrow Q \wedge R$	P
3.	$\sim S$	P
4.	$\sim(P \vee \sim T)$	PP 1, 3
5.	$\sim P \wedge T$	DL 4
6.	T	S 5
7.	$Q \wedge R$	PP 2, 6
8.	$R \wedge Q$	CL 7

(B4) Prove: $\sim P$

1.	$R \rightarrow \sim P$	P
2	$(R \wedge S) \vee T$	P
3.	$T \rightarrow (Q \vee U)$	P
4.	$\sim Q \wedge \sim U$	P
5.	$\sim(Q \vee U)$	DL 4
6.	$\sim T$	TT 5, 3
7.	$R \wedge S$	TP 2, 6
8.	R	S 7
9.	$\sim P$	PP 1, 8

Ⓒ F : obj floats on H_2O
L : obj less dense than H_2O
W : You can walk on H_2O
D : obj can displace own wt

Prove: $F \leftrightarrow L$

1.	$\neg(F \rightarrow L) \rightarrow W$	P
2.	$\neg W$	P
3.	$L \rightarrow D$	P
4.	$D \rightarrow F$	P
5.	$F \rightarrow L$	TT 1, 2
6.	$L \rightarrow F$	HS 3, 4
7.	$F \leftrightarrow L$	LB 5, 6

D1

$P: 2 \times 5 = 5 + 5$

$Q: 2 \times 4 = 4 + 4$

Prove: $P \to Q$

1.	$Q \leftrightarrow P$	P
2.	$P \to Q$	LB 1

D2

$P: x = 4$

$Q: 3x + 2 = 14$

$R: 3x = 12$

Prove: $P \leftrightarrow Q$

1.	$Q \leftrightarrow R$	P
2.	$R \leftrightarrow P$	P
3	$Q \to R$	LB 1
4	$R \to P$	LB 2
5	$Q \to P$	HS 3,4
6.	$P \to R$	LB 2
7.	$R \to Q$	LB 1
8.	$P \to Q$	HS 6,7
9.	$P \leftrightarrow Q$	LB 5,8

D3

$P: x + y = 5$

$Q: 3x + y = 11$

$R: 3x = 9$

$S: y = 2$

Prove: P

1.	$Q \leftrightarrow R$	P
2.	$(R \to Q) \leftrightarrow S$	P
3	$\sim S \vee P$	P
4.	$R \to Q$	LB 1
5.	$(R \to Q) \to S$	LB 2
6	S	PP 4,5
7.	P	TP 3,6

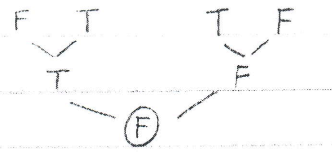
P, Q] true   A, B] False

Lily 6

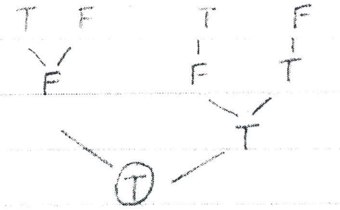
Logic HW

Ⓐ

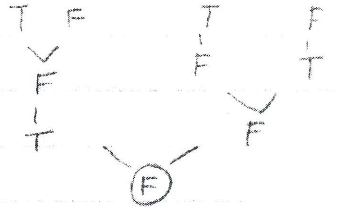
1) $(A \to P) \to (P \to A)$



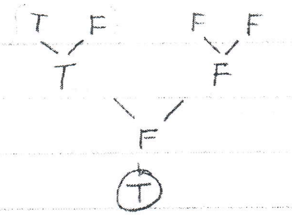
2) $(P \to A) \to (\sim P \to \sim A)$



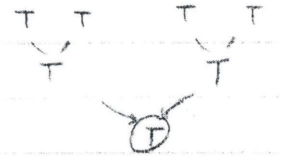
3) $\sim(P \land B) \to (\sim P \land \sim B)$



4) $\sim[(P \lor B) \land (B \lor A)]$



5) $(P \to Q) \leftrightarrow (Q \to P)$



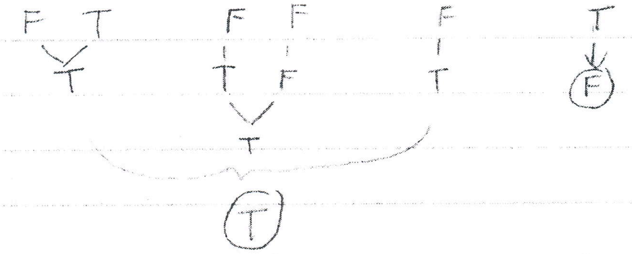
Ⓑ

1) 4 $(T \land S) \to R$	LS 1	2) 6. $\sim Q$	TT 1,5
5 $\sim(T \land S)$	TT 2,4	7. $\sim P$	TT 2,6
6 $\sim T \lor \sim S$	DL 5	8 T	TP 3,7
7 $\sim S$	TP 3,6	9. S	PP 4,8

3)

T	Q	R	$T \to Q$	$\sim T \lor R$	$\sim R$	$\sim Q$
F	T	F	T	T	T	$\boxed{F}$

$(T \to Q) \land (\sim T \lor R) \land (\sim R) \to \sim Q$

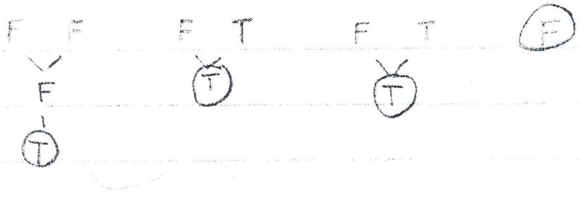


True	False
Q	R
	T

4)

P	Q	R	T	$\sim(P \lor Q)$	$P \lor R$	$T \to R$	T
F	F	T	F	T	T	T	$\boxed{F}$

$\sim(P \lor Q) \land (P \lor R) \land (T \to R) \to T$



True	False
R	T
	P
	R

Ⓒ

4	P	S 1
5	Q	S 1
6	$\sim R$	PP 2,4
7	$\sim S$	PP 3,5
8	$\sim R \wedge \sim S$	A 6,7
9	$\sim(R \vee S)$	DL 8

Ⓓ Hypothetical Syllogism

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	F	F
F	[illegible]	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

All cases where
prem — Ⓣ
concl also Ⓣ

Ⓔ

P	Q	$\sim(P \vee Q)$	$\sim P \wedge \sim Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

↑
same as

HW 7
Logic

Ⓐ

1) $\sim P \to Q$

1. $P \vee Q$	P
2. $\sim P$	P
3. Q	TP 1, 2
4 $\sim P \to Q$	CP 2, 3

2. $C \to \sim D$

1. $B \to \sim C$	P
2. $\sim(D \wedge \sim B)$	P
3. $\sim D \vee B$	DL 2
4. C	P
5. $\sim B$	TT 1, 4
6 $\sim D$	TP 3, 5
7 $C \to \sim D$	CP 4, 6

3) $P \to P \wedge Q$

1. $R \to T$	P
2. $T \to \sim S$	P
3 $(R \to \sim S) \to Q$	P
4. $R \to \sim S$	HS 1, 2
5 Q	PP 3, 4
6. P	P
7. $P \wedge Q$	A 5, 6
8 $P \to P \wedge Q$	CP 6, 7

4) $S \to Q$

1. $R \to Q$	P
2. $T \to R$	P
3. $S \to T$	P
4. $S \to R$	HS 3, 2
5. $S \to Q$	HS 4, 1

5) $\sim(R \wedge S) \to T$

1. $\sim P$	P
2. $\sim R \to T$	P
3. $\sim S \to P$	P
4. S	TT 1, 3
5. $\sim(R \wedge S)$	P
6. $\sim R \vee \sim S$	DL 5
7. $\sim R$	TP 4, 6
8. T	PP 7, 2
9 $\sim(R \wedge S) \to T$	CP 5, 8

⑥ $T \vee \sim S \to R$

1. $\sim R \to Q$	P
2. $T \to \sim Q$	P
3. $\sim S \to \sim Q$	P
4 $T \vee \sim S$	P
5 $\sim Q \vee \sim Q$	DS 2, 3, 4
6. $\sim Q$	DP 5
7 R	TT 6, 1
8. $T \vee \sim S \to R$	CP 4, 7

7) $\sim Q \to T \wedge S$

1. $R \to S$	P
2 $S \to Q$	P
3. $R \vee (S \wedge T)$	P
4. $R \to Q$	HS 1, 2
5. $\sim Q$	P
6. $\sim R$	TT 4, 5
7. $S \wedge T$	TP 3, 6
8. $T \wedge S$	CL 7
9 $\sim Q \to T \wedge S$	CP 5, 8

8) $(P \wedge Q) \to (S \wedge T)$

1. $R \vee S$	P
2. $\sim T \to \sim P$	P
3. $R \to \sim Q$	P
4 $P \wedge Q$	P
5. P	S 4
6. Q	S 4
7. T	TT 5, 2
8. $\sim R$	TT 6, 3
9. S	TP 1, 8
10. $S \wedge T$	A 7, 9
11. $P \wedge Q \to S \wedge T$	CP 4, 10

9) $S \to P \vee Q$

1. $S \to T$	P
2. $R \to P$	P
3. $T \to R$	P
4. $T \to P$	HS 3, 2
5. S	P
6. T	PP 1, 5
7. P	PP 4, 6
8. $P \vee Q$	LA 7
9. $S \to P \vee Q$	CP 5, 8

10) $\sim(P \vee R) \to T$

1 $Q \to P$	P
2 $T \vee S$	P
3 $Q \vee \sim S$	P
4 $\sim(P \vee R)$	P
5. $\sim P \wedge \sim R$	DL 4
6. $\sim P$	S 5
7 $\sim Q$	TT 6, 1
8. $\sim S$	TP 3, 7
9. T	TP 2, 8

A : $x = 0$

B : $x = 1$

C : $x^3 - 3x^2 + 2x = 0$

D : $x^2 - x = 0$

E : $x = 2$

11) $E \rightarrow K$

1.	$E \vee F \rightarrow G$	P
2.	$J \rightarrow \sim G \wedge \sim H$	P
3.	$J \vee K$	P
4	E	P
5.	$E \vee F$	LA 4
6.	G	PP 1,5
7.	$G \vee H$	LA 7
8	$\sim(\sim G \wedge \sim H)$	DL 7
9.	$\sim J$	TT 8, 2
10.	K	TP 3,9
11	$E \rightarrow K$	CP 4,10

12) $Q \leftrightarrow \sim P$

1.	$\sim(\sim P \wedge \sim Q)$	P
2.	$S \rightarrow \sim Q$	P
3	$\sim P \vee S$	P
4	$P \vee Q$	DL 1
5.	Q	P
6.	$\sim S$	TT 2,5
7	$\sim P$	TP 3,6
8	$Q \rightarrow \sim P$	CP 5,7
9.	$\sim P$	P
10.	Q	TP 4,9
11	$\sim P \rightarrow Q$	CP 9,10
12.	$Q \leftrightarrow \sim P$	LB 8,11

13) $P \rightarrow (Q \rightarrow R)$

1.	$P \wedge Q \rightarrow R$	P
2.	P	P
3	Q	P
4	$P \wedge Q$	A 2,3
5.	R	PP 1,4
6.	$Q \rightarrow R$	CP 3,5

14) $A \vee B \rightarrow C$

1.	$A \rightarrow D$	P
2.	$B \rightarrow D$	P
3.	$E \vee D \rightarrow C$	P
4.	$A \vee B$	P
5.	$D \vee D$	DS 1,2,4
6.	D	DP 5
7.	$E \vee D$	LA 6
8.	C	PP 3,7
9.	$A \vee B \rightarrow C$	CP 4,8

B 15) $T \rightarrow E$

1.	$\sim T \vee B$	P
2.	$B \rightarrow U$	P
3.	$U \rightarrow E$	P
4	$B \rightarrow E$	HS 2,3
5.	T	P
6.	B	TP 1,5
7.	E	PP 4,6
8.	$T \rightarrow E$	CP 5,7

①

HW 8

Logic

1) ~P

1.	~(P∧Q)	P
2.	P→R	P
3.	Q∨~R	P
4.	~P∨~Q	DL 1
5.	P	P
6.	~Q	TP 4,5
7	~R	TP 3,6
8.	~P	TT 2,7
9.	P∧~P	A 5,8
10.	~P	RAA 5,9

2) ~T

1.	T→~S	P
2.	F→~T	P
3.	S∨F	P
4.	T	P
5.	~S	PP 1,4
6.	F	TP 3,5
7.	~T	PP 2,6
8.	T∧~T	A 4,7
9.	~T	RAA 4,8

3) R

1.	~(P∧Q)	P
2.	~R→Q	P
3.	~P→R	P
4.	~P∨~Q	DL 1
5.	~R	P
6.	Q	PP 2,5
7.	P	TT 5,3
8.	~Q	TP 7,4
9.	R	TT 2,8
10.	~R∧R	A 5,9

4) ~(A∧D)

1.	A→B∨C	P
2.	B→~A	P
3.	D→~C	P
4	A∧D	P
5.	A	S4
6.	D	S4
7.	B∨C	PP 5,1
8.	~C	PP 3,6
9.	B	TP 7,8
10.	~A	PP 2,9
11.	A∧~A	A 5,10
12.	~(A∧D)	RAA 4,11

5) ~E∨M

1.	S∨D	P
2.	S→~E	P
3.	D→M	P
4	~E∨M	DS 1,2,3

6) ~T

1,	P∨Q	P
2,	T→~P	P
3.	~(Q∨R)	P
4.	~Q∧~R	DL 3
5.	T	P
6	~P	PP 5,2
7	Q	TP 6,1
8	~Q	S 4
9.	Q∧~Q	A 7,8
10.	~T	RAA 5,9

(12) ~S∨~T

7) ~(T∨S)

1.	~R∨~B	P
2.	T∨S→R	P
3.	B∨~S	P
4.	~T	P
5.	T∨S	P
6.	S	TP 4,5
7.	R	PP 2,5
8.	~B	TP 7,1
9.	~S	TP 8,3
10.	S∧~S	A 6,9
11.	~(T∨S)	RAA 5,10

8) ~P

1.	P→~S	P
2.	S∨~R	P
3.	~(T∨~R)	P
4.	~T∧R	DL 3
5.	R	S 4
6.	S	TP 5,2
7.	~P	TT 6,1

9) ~S∨~T

1.	~P→~S	P
2.	~P∨R	P
3.	R→~T	P
4.	~(~S∨~T)	P
5.	S∧T	DL 4
6.	S	S 5
7.	P	TT 6,1
8.	R	TP 7,2
9.	~T	PP 3,8
10.	T	S 5

Ⓑ 5, 6, 8

10) R

1.	$T \wedge R \leftrightarrow \sim S$	P
2.	$\sim S \rightarrow T$	P
3.	$\sim R \rightarrow \sim S$	P
4.	$\sim R \rightarrow T$	HS 3, 2
5.	$\sim S \rightarrow T \wedge R$	LB 1
6.	$T \wedge R \rightarrow \sim S$	LB 1
7.	$\sim R$	P
8.	T	PP 4, 7
9	$\sim S$	PP 7, 3
10.	$T \wedge R$	PP 9, 5
11.	R	S 10
12.	$R \wedge \sim R$	A 7, 11
13.	R	RAA 7, 12

11) $A: y=1$ $B: x^2 > xy$

$C: x=1$ $D: x+y=y$

$E: x>y$

$\sim(A \rightarrow \sim B)$

1	$C \vee \sim(D \vee \sim E)$	P
2	$E \rightarrow B \wedge A$	P
3	$\sim C$	P
4	$\sim(D \vee \sim E)$	TP 1, 3
5	$\sim D \wedge E$	DL 4
6.	E	S 5
7	$B \wedge A$	PP 6, 2
8	$A \rightarrow \sim B$	P
9	B	S 7
10	$\sim A$	TT 8, 9
11	A	S 7
12	$\sim A \wedge A$	A 10, 11
13	$\sim(A \rightarrow \sim B)$	RAA 8, 12

Day 1
Logic
HW

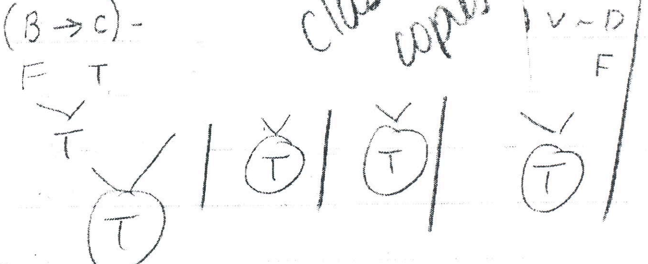
Ⓐ

1)

1.	A → C	P
2.	~(B ∧ C)	P
3.	A ∨ (B ∧ C)	P
4.	B	P
5.	~B ∨ ~C	DL 2
6.	~C	TP 4,5
7.	~A	TT 1,6
8.	B ∧ C	TP 7,3
9.	~(B∧C) ∧ (B∧C)	A 2,8

Contradiction!
∴ incosistent premises

make class set copies



T	F
A	D
C	B

consistent

Ⓑ ①

Prove L ∨ E

1	L → E	P
2.	~L → ~E	P

(L→E) ∧ (~L→~E) → L∨E

L	E	L → E	~L	~E	~L → ~E	(L→E) ∧ (~L→~E)	L∨E	$\overrightarrow{\text{(concl)}}$
T	T	T	F	F	T	T	T	T
T	F	F	F	T	T	F	T	T
F	T	T	T	F	F	F	T	T
F	F	T	T	T	T	T	F	F

Invalid

2) (E → P) ∧ (~E → Y) → (E ∨ Y)

E	P	Y	E → P	~E → Y	(E→P) ∧ (~E→Y)	E ∨ Y	$\overrightarrow{\text{(concl)}}$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	F	F	F	T

Valid

c) HS : $(P \to Q) \wedge (Q \to R) \longrightarrow (P \to R)$

P	Q	R	$P \to Q$	$Q \to R$	$(P\to Q) \wedge (Q \to R)$	$P \to R$	(conclu)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

valid arg

D) 1) $P \vee \sim Q \to (P \to \sim Q)$

P	Q	$\sim Q$	$P \vee \sim Q$	$\sim Q$	$P \to \sim Q$	$\to$
T	T	T	T	F	F	F
T	F	T	T	T	T	T
F	T	F	F	F	T	T
F	F	T	T	T	T	T

not taut

2) $\sim P \vee Q \to (P \to Q)$

P	Q	$\sim P$	$\sim P \vee Q$	$P \to Q$	$\sim P \vee Q \to (P \to Q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

taut

3) $P \wedge Q \to P \vee R$

P	Q	$P \wedge Q$	$P \vee R$	$\to$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

taut

Ⓕ2

R	S	T	$T \vee \sim R$	$\sim (R \to S)$	$T \to S$
T	T	T	T	F	T
T	T	F	F	F	T
T	F	T	T	T	F
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	F	T
F	F	T	T	F	F
F	F	F	T	F	T

inconsistent ; prem are
never all true

Day 9
HW
(Cont)

D)

4) $P \wedge Q \rightarrow (P \leftrightarrow Q \vee R)$

P	Q	R	$P \wedge Q$	$Q \vee R$	$P \leftrightarrow Q \vee R$	$\rightarrow$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	T	T
T	F	F	F	F	F	T
E	T	T	F	T	F	T
F	T	F	F	T	F	T
F	F	T	F	T	F	T
F	F	F	F	F	T	T

taut!

E) 1) $P \vee \sim Q \rightarrow \sim P \overset{?}{\equiv} P \rightarrow \sim P \wedge Q$

P	Q	$\sim Q$	$P \vee \sim Q$	$\sim P$	$\rightarrow$	P	$\sim P \wedge Q$	$\rightarrow$
T	T	F	T	F	F	T	F	F
T	F	T	T	F	F	T	F	F
F	T	F	F	T	T	F	T	T
F	F	T	T	T	T	F	F	T

yes. logically equiv

2) $\sim(A \rightarrow \sim B) \rightarrow C \overset{?}{\equiv} B \wedge \sim C \rightarrow \sim A$

A	B	C	$\sim B$	$A \rightarrow \sim B$	$\sim(A \rightarrow \sim B)$	$\rightarrow$	$B \wedge \sim C$	$\sim A$	$\rightarrow$
T	T	T	F	F	T	T	F	F	T
T	T	F	F	F	T	F	T	F	F
T	F	T	T	T	F	T	F	F	T
T	F	F	T	T	F	T	F	F	T
F	T	T	F	T	F	T	F	T	T
F	T	F	F	T	F	T	T	T	T
F	F	T	T	T	F	T	F	T	T
F	F	F	T	T	F	T	F	T	T

Yes! Logic - equiv

Inconsistent

F) ①
5. T S2
6 R S2
7. P PP 5, 1
8. PVS LA 7
9. Q PP 8, 4
10 ~Q TT 3, 6
11 $Q \wedge \sim Q$ A 9, 10

4) 1) Prove $\sim P$

1	$\sim(P\wedge Q)$	P
2	$P\rightarrow R$	P
3.	$Q\vee\sim R$	P
4.	$\sim P\vee\sim Q$	DL 1
5.	P	P
6.	R	PP 2,5
7	Q	TP 3,6
8.	$\sim Q$	TP 4,5
9	$Q\wedge\sim Q$	A 7,8
10.	$\sim P$	RAA 5,9

2) $\sim(A\wedge D)$

1)	$A\rightarrow B\vee C$	P
2)	$B\rightarrow\sim A$	P
3)	$D\rightarrow\sim C$	P
4)	$A\wedge D$	P
5)	A	S 4
6)	D	S 4
7)	$\sim B$	TT 5,2
8)	$\sim C$	PP 3 6
9)	$B\vee C$	PP 1,5
10)	C	TP 7,9
11)	$C\wedge\sim C$	A 8,10
12)	$\sim(A\wedge D)$	RAA 4,11

3) $\sim P$

1	$P\rightarrow\sim S$	P
2	$S\vee\sim R$	P
3	$\sim(T\vee\sim R)$	P
4	$\sim T\wedge R$	DL 3
5	R	S 4
6	S	TP 2,5
7	P	P
8	$\sim S$	PP 1,7
9	$S\wedge\sim S$	A 6,8
10	$\sim P$	RAA 7,9

4) $\sim S\vee\sim T$

1)	$\sim P\rightarrow\sim S$	P
2)	$\sim P\vee R$	P
3)	$R\rightarrow\sim T$	P
4)	$\sim(\sim S\vee\sim T)$	P
5)	$S\wedge T$	DL 4
6)	S	S 5
7)	T	S 5
8)	P	TT 6,1
9)	R	TP 8,2
10)	$\sim T$	PP 3,9
11)	$T\wedge\sim T$	A 7,10
12)	$\sim S\vee\sim T$	RA4 4,11

Ⓗ $(C\wedge\sim D)\wedge(C\rightarrow\sim A)\wedge(D\vee\sim B)\longrightarrow\sim(A\vee B)$

Ⓘ Prove $S\rightarrow Q\wedge\sim T$

Given 1) $Q\rightarrow T\vee R$

2) $\sim S$

3) $R\vee T\rightarrow S$

Logic Review Sheet Name ____________________

1. Use a CONDITIONAL proof to prove the following:

Prove: ~R → K

1) Q → K P
2) R V Q P
3)
4)
5)
6)

2. Use and INDIRECT proof to prove the following:

Prove: ~(A → B)

1) ~(~A V B) P
2)
3)
4)
5)
6)
7)
8)

3.

Prove: P

1) ~S V (~G → P) P
2) ~ (G & ~D) P
3) S P
4) D → ~S P
5)
6)
7)
8)
9)

4. Determine the truth value of the following sentences if

A = "2 + 6 = 8"
B = "2 – 5 = 3"
C = "3 X 4 = 12"
D = "5 x 0 = 5"

1) (A & B) & (C & D) → A V D

2) (~A → B) → (D → C)

5. Prove that the following set of premises are *inconsistent* by deriving a contradiction from each.

(1) Q → P
(2) ~(P V R)
(3) Q V R

6. Show by a truth table which of the following examples of inference is valid. Examine the entire truth table and write the words 'valid' or 'invalid' beside each.

A. If Becky is late then Christine is early.
If Becky is not late then Christine is not early.
Therefore, either Becky is late or Christine is early.

B. (P V Q) & ~Q
Therefore, P.

7. If P and Q are distinct atomic sentences, which of the following are tautologies? Use truth tables and state whether it is a tautology or not.

A. (P → Q) ↔ (Q→ P)

B. ~P V ~Q → (P → Q)

8. If any of the symbolized arguments below are valid, show a derivation of the conclusion by means of a complete formal proof. If they are invalid, prove it by a truth assignment.

A. Prove: A
(1) ~(P V Q)
(2) P V R
(3) A → R

B. Prove: R V ~Q
(1) S & ~A
(2) A→ P
(3) S → R
(4) ~P → ~Q